FIVE MARKS QUESETIONS

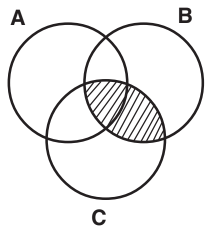
1. SETS AND FUNCTIONS

1. Use Venn diagrams to verify $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

Solution :

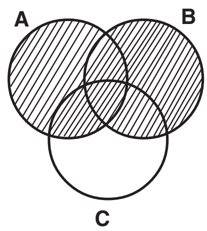
LHS:

$B \cap C$

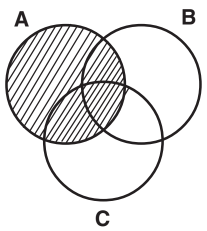


RHS

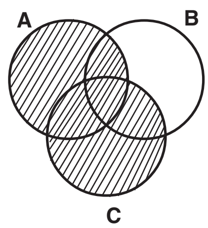
$A \cup B$



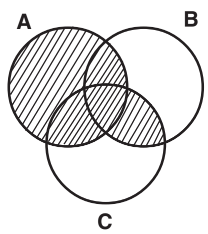
A



$A \cup C$

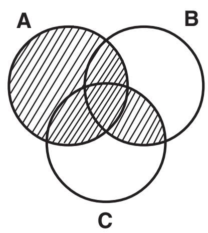


$A \cup (B \cap C)$



------ (I)

$(A \cup B) \cap (A \cup C)$



--------- (II)

I = II

LHS = RHS

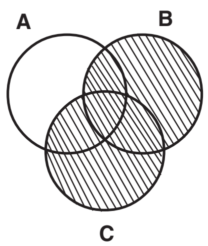
$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

2. Use Venn diagrams to verify $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

Solution:

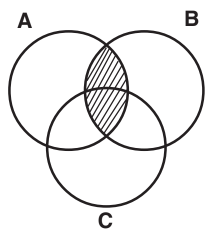
LHS

$B \cup C$

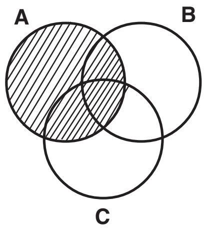


RHS

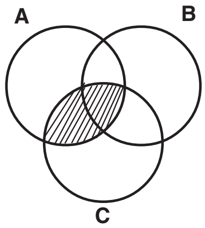
$A \cap B$



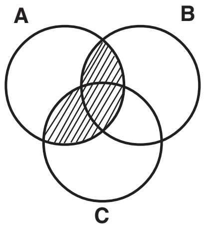
A



$A \cap C$

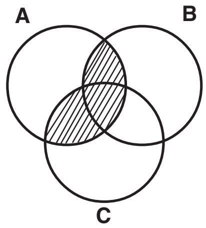


$A \cap (B \cup C)$



------ (I)

$(A \cap B) \cup (A \cap C)$



-------- (II)

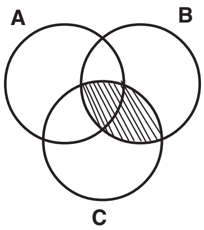
$$I = II$$
$$LHS = RHS$$
$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C).$$

3. Use Venn diagrams to verify $A \setminus (B \cap C) = (A \setminus B) \cup (A \setminus C)$.

Solution:

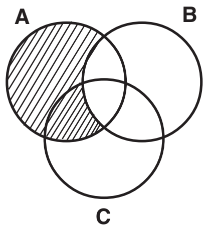
LHS

$B \cap C$

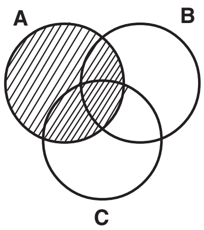


RHS

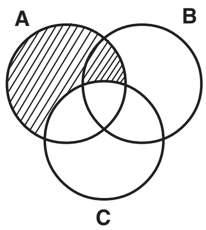
$A \setminus B$



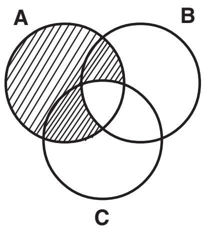
A



$A \setminus C$

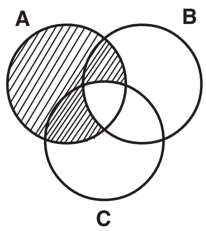


$A \setminus (B \cap C)$



------ (I)

$(A \setminus B) \cup (A \setminus C)$



-------- (II)

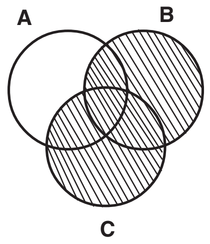
$$I = II$$
$$LHS = RHS$$
$$A \setminus (B \cap C) = (A \setminus B) \cup (A \setminus C)$$

4. Use Venn diagrams to verify $A \backslash (B \cup C) = (A \backslash B) \cap (A \backslash C)$

Solution:

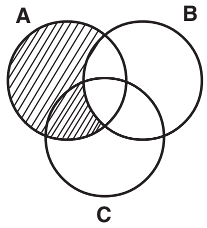
LHS

$(B \cup C)$

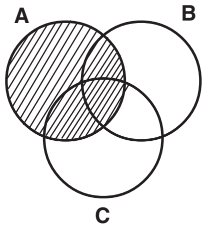


RHS

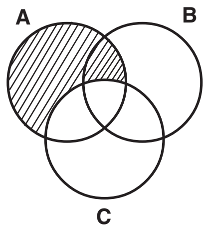
$(A \backslash B)$



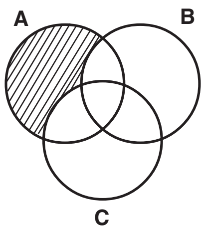
A



$(A \backslash C)$

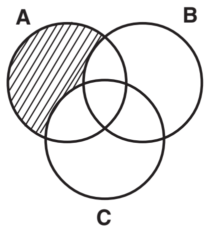


$A \backslash (B \cup C)$



---- (I)

$(A \backslash B) \cap (A \backslash C)$



------ (II)

I = II

LHS = RHS

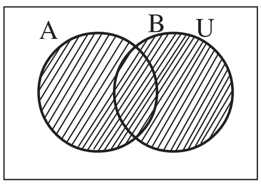
$A \backslash (B \cup C) = (A \backslash B) \cap (A \backslash C)$

5. Use Venn diagrams to verify De Morgan's law for complementation $(A \cup B)' = A' \cap B'$ ★

Solution :

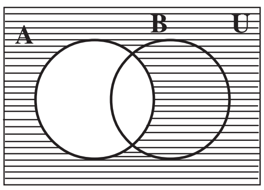
LHS

$A \cup B$

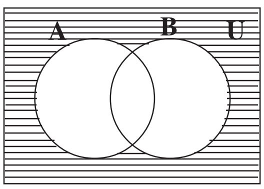


RHS

$A' = U - A$

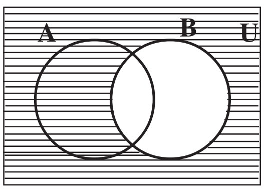


$(A \cup B)' = U - A \cup B$

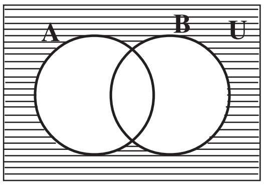


--- (I)

$B' = U - B$



$A' \cap B'$



-------- (II)

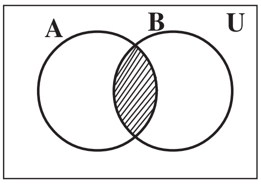
$$I = II$$
$$LHS = RHS$$
$$(A \cup B)' = A' \cap B'$$

6. Use Venn diagrams to verify $(A \cap B)' = A' \cup B'$

Solution:

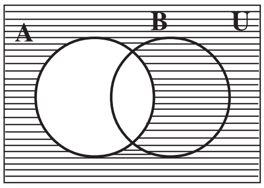
LHS

$A \cap B$

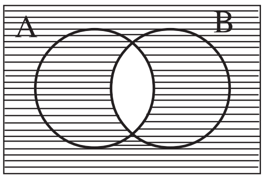


RHS

$A' = U - A$

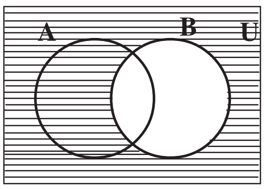


$(A \cap B)' = U - A \cap B$

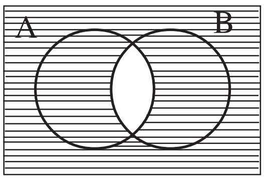


---- (I)

$B' = U - B$



$A' \cup B'$



-------- (II)

$$I = II$$
$$LHS = RHS$$
$$(A \cap B)' = A' \cup B'$$

7. U = {-2, -1, 0, 1, 2, 3, ... 10}, A = {-2, 2, 3, 4, 5} B = {1, 3, 5, 8, 9} P.T.

i) $(A \cup B)' = A' \cap B'$ ii) $(A \cap B)' = A' \cup B'$

i) **L.H.S.** = $(A \cup B)'$

$A \cup B$ = {-2, 2, 3, 4, 5} $\cup$ {1, 3, 5, 8, 9}

= {-2, 1, 2, 3, 4, 5, 8, 9}

$(A \cup B)'$ = U \ $(A \cup B)$

= {-2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10} \ {-2, 1, 2, 3, 5, 8, 9}

= {-1, 0, 6, 7, 10} ------- (I)

R.H.S. $= A' \cap B'$

$A' = U \setminus A$

$= \{-2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} \setminus \{-2, 2, 3, 4, 5\}$

$= \{-1, 0, 1, 6, 7, 8, 9, 10\}$

$B' = U \setminus B$

$= \{-2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} \setminus \{1, 3, 5, 8, 9\}$

$= \{-2, -1, 0, 2, 4, 6, 7, 10\}$

$A' \cap B' = \{-1, 0, 1, 6, 7, 8, 9, 10\} \cap \{-2, -1, 0, 2, 4, 6, 7, 10\}$

$= \{-1, 0, 6, 7, 10\}$ ----- (II)

I = II

$(A \cup B)'\ A' \cap B'$

ii) $(A \cup B)' = A' \cup B'$

L.H.S. $= (A \cap B)'$

$(A \cap B) = \{-2, 2, 3, 4, 5\} \cap \{1, 3, 5, 8, 9\}$

$= \{3, 5\}$

$(A \cap B)' = U \setminus (A \cap B)$

$= \{-2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} \setminus \{3, 5\}$

$= \{-2, -1, 0, 1, 2, 4, 6, 7, 8, 9, 10\}$ ----- (I)

R.H.S. $= A' \cup B'$

$A' = U \setminus A$

$A' = \{-2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} \setminus \{-2, 2, 3, 4, 5\}$

$= \{-1, 0, 1, 6, 7, 8, 9, 10\}$

$B' = U \setminus B$

$B' = \{-2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} \setminus \{1, 3, 5, 8, 9\}$

$= \{-2, -1, 0, 2, 4, 6, 7, 10\}$

$A' \cup B' = \{-1, 0, 1, 6, 7, 8, 9, 10\} \cup \{-2, -1, 0, 2, 4, 6, 7, 10\}$

$= \{-2, -1, 0, 1, 2, 4, 6, 7, 8, 9, 10\}$ ----- (II)

I = II

$(A \cap B)' = A' \cup B'$

8. Let $A = \{a, b, c, d, e, f, g, x, y, z\}$, $B = \{1, 2, c, d, e\}$ and $C = \{d, e, f, g, z, y\}$ P.T. $A \setminus (B \cup C) = (A \setminus B) \cap (A \setminus C)$

Solution :

$B \cup C = \{1, 2, c, d, e\} \cup \{d, e, f, g, z, y\}$

$= \{1, 2, c, d, e, f, g, y\}$

$A \setminus B \cup C = \{a, b, c, d, e, f, g, x, y, z\} \setminus \{1, 2, c, d, e, f, g, y\}$

$= \{a, b, x, z\}$ ---- (I)

$A \setminus B = \{a, b, c, d, e, f, g, x, y, z\} \setminus \{1, 2, c, d, e\}$

$= \{a, b, f, g, x, y, z\}$

$A \setminus C = \{a, b, c, d, e, f, g, x, y, z\} \setminus \{d, e, f, g, z, y\}$

$= \{a, b, c, x, z\}$

$(A \setminus B) \cap (A \setminus C) = \{a, b, f, g, x, y, z\} \cap \{a, b, c, x, z\}$

$= \{a, b, x, z\}$ ---- (I)

I = II

$A \setminus (B \cup C) = (A \setminus B) \cap (A \setminus C)$

9. In a town 85% of the people speak Tamil, 40% speak English and 20% speak Hindi. Also 32% speak English and Tamil, 13% speak Tamil and Hindi 10% speak English and Hindi. Find the percentage of people who can speak all the three languages.

Tamil - T

English - E

Hindi - H

No. of people who can speak Tamil n(T) = 85%

No. of people who can speak English n(E) = 40%

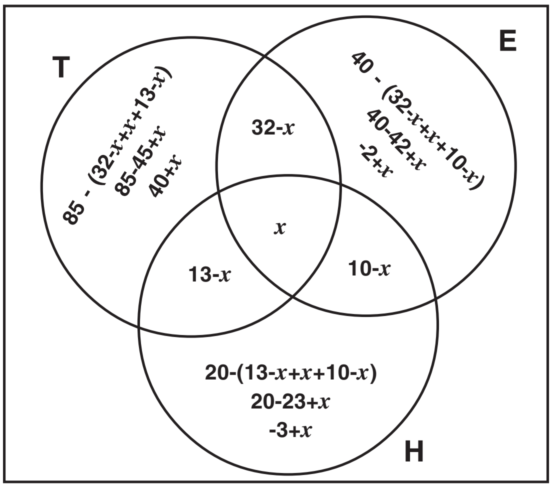
No. of people who can speak Hindi n(H) = 20%

$n(T \cap E) = 32\%$

$n(T \cap H) = 13\%$

$n(E \cap H) = 10\%$

$n(T \cap E \cap H) = x$



$40 + x + 32 - x + 13 - x + x - 2 + x - 3 + x + 10 - x = 100$

$95 - 5 + x = 100$

$90 + x = 100$

$x = 100 - 90$

$x = 10\%$

No. of people who can speak all the three languages = 10%

10. An advertising agency finds that, of its 170 clients, 115 use Television, 110 use Radio and 130 use Magazines. Also, 85 use Television and Magazines, 75 use Television and Radio, 95 use Radio and Magazines, 70 use all the three. Draw Venn diagram to represent these data. Find
(i) how many use only Radio? (ii) how many use only Television?
(iii) how many use Television and magazine but not radio?

Television = T

Radio = R

Magazine = M

$n(T) = 115$

$n(R) = 110$

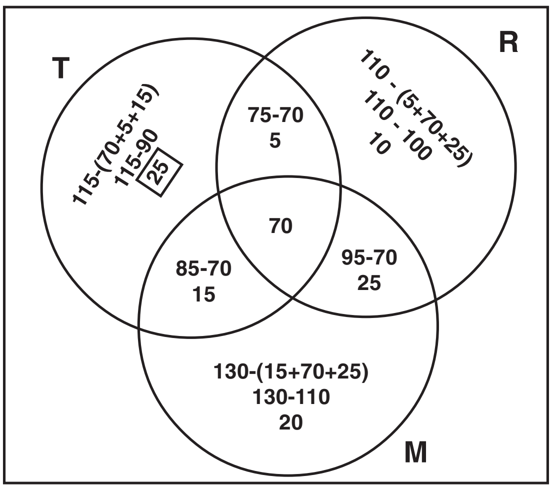
$n(M) = 130$

$n(T \cap M) = 85$

$n(T \cap R) = 75$

$n(R \cap M) = 95$

$n(T \cap R \cap M) = 70$



i) No. of clients using only Radio = 10

ii) No.of clients using only T.V. = 25

iii) No. of clients using T.V. and magazines but not radio = 15

11. A function f : [-3, 7) $\rightarrow$ R is defined by as follows

$$f(x) = \begin{cases} 4x^2 - 1 & : \quad -3 \le x < 2 \\ 3x - 2 & : \quad 2 \le x \le 4 \\ 2x - 3 & : \quad 4 < x < 7 \end{cases}$$ find (i) f(5) + f(6)

ii) f(1) - f(-3) (iii) f(-2) - f(4) (iv) $\frac{f(3)+f(-1)}{2f(6)-f(1)}$

Solution:

$$f(x) = \begin{cases} 4x^2 - 1 & : \quad -3 \le x < 2 \quad (-3,-2,-1,0,1) \\ 3x - 2 & : \quad 2 \le x \le 4 \quad (2,3,4) \\ 2x - 3 & : \quad 4 < x < 7 \quad (5,6) \end{cases}$$

i) f(5) + f(6) =?

f(x) = 2x - 3

f(5) = 2 x 5 - 3

= 10 - 3

f(5) = 7

f(6) = 2 $\times$ 6 - 3

= 12 - 3

f(6) = 9

f(5) + f(6) = 7 + 9

f(5) + f(6) = 16

ii) f(1) - f(-3) = ?

f(x) = $4x^2$ - 1

f(1) = 4 x 1^2 - 1

= 4 - 1

f(1) = 3

f(-3) = 4 x $(-3)^2$ - 1

= 4 x 9 - 1

= 36 - 1

f(-3) = 35

f(1) - f(-3) = 3 - 35

f(1) - f(-3) = -32

iii) f(-2) - f(4)

f(x) = $4x^2$ - 1

f(-2) = 4x $(-2)^2$ - 1

= 4 x 4 - 1

= 16 - 1

f(-2) = 15

f(x) = 3x - 2

f(4) = 3 x 4 - 2

= 12 - 2

f(4) = 10

$f(-2) - f(4) = 15 - 10$

$f(-2) - f(4) = 5$

iv) $\frac{f(3)+f(-1)}{2f(6)-f(1)} = ?$

$f(3) + f(-1)$

$f(x) = 3x - 2$

$f(3) = 3 \times 3 - 2$

$= 9 - 2$

$f(3) = 7$

$f(x) = 4x (-1)^2 - 1$

$= 4 - 1$

$f(-1) = 3$

$f(3) + f(-1) = 7 + 3$

$f(3) + f(-1) = 10$

$2f(6) - f(-1) = 10$

$2f(6) - f(1) = ?$

$2(x) = 2x - 3$

$f(6) = 2 \times 6 - 3$

$= 12 - 3$

$f(6) = 9$

$2f(6) = 18$

$f(1) = 4 \times 1^2 - 1$

$= 4 - 1$

$f(1) = 3$

$2f(6) - f(1) = 18 - 3$

$= 15$

$\frac{f(3)+f(-1)}{2f(6)-f(1)} = \frac{10}{15}$

Ans: $\frac{2}{3}$

12. Let A= { 0, 1, 2, 3 } and B = { 1, 3, 5, 7, 9 } be two sets. Let $f : A \rightarrow B$ be a function given by f(x)=2x+1. Represent this function as (i) a set of ordered pairs (ii) a table (iii) an arrow diagram and (iv) a graph.

Solution

$f(x) = 2x + 1$

$f(0) = 2 \times 0 + 1 = 0+1 = 1$

$f(1) = 2 \times 1 + 1 = 2 + 1 = 3$

$f(2) = 2 \times 2 + 1 = 4 + 1 = 5$

$f(3) = 2 \times 3 + 1 = 6 + 1 = 7$

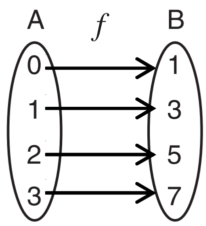
(i) Set of ordered pairs

{ (0, 1), (1, 3), (2, 5), (3, 7) }

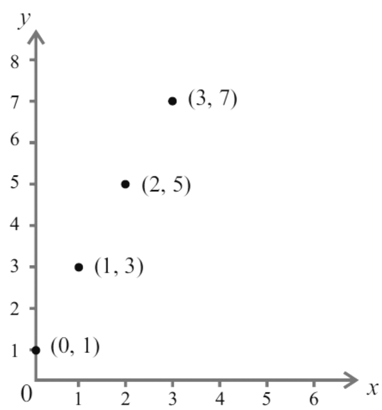
(ii) Table form

x	0	1	2	3
f (x)	1	3	5	7

(iii) Arrow Diagram



(iv) Graph



13. A function f : [1, 6) → R is defined as follows

$$f(x) = \begin{cases} 1+x & 1 \le x < 2 \\ 2x-1 & 2 \le x < 4 \\ 3x^2-10 & 4 \le x < 6 \end{cases}$$

Find the value of (i) f(5) (ii) f (3) (iii) f (1) (iv) f(2)- f(4) (v) 2 f(5) - 3f (1).

Solution:

$$f(x) = \begin{cases} 1+x & 1 \le x < 2 & (1) \\ 2x-1 & 2 \le x < 4 & (2, 3) \\ 3x^2-10 & 4 \le x < 6 & (4, 5) \end{cases}$$

i) $f(x) = 3x^2 - 10$

$f(5) = 3 \times 5^2 - 10$

$= 3 \times 25 - 10$

$= 75 - 10$

$f(5) = 65$

ii) $f(x) = 2x - 1$

$f(3) = 2 \times 3 - 1$

$= 6 - 1$

$f(3) = 5$

iii) $f(x) = 1 + x$

$f(1) = 1 + 1$

$f(1) = 2$

iv) f(2) - f(4)

$f(x) = 2x - 1$

$f(2) = 2 \times 2 - 1$

$= 4 - 1$

$f(2) = 3$

$f(x) = 3x^2 - 10$

$f(4) = 3 \times 4^2 - 10$

$= 3 \times 16 - 10$

$= 48 - 10$

$f(4) = 38$

$f(2) - f(4) = 3 - 38$

$f(2) - f(4) = -35$

v) 2f(5) - 3f(1)

$2f(5) = 2 \times 65$

$= 130$

$3f(1) = 3 \times 2$

$= 6$

$2f(5) - 3f(1) = 130 - 6$

$2f(5) - 3f(1) = 124$

14. Let A = {4, 6, 8, 10} and B = {3, 4, 5, 6, 7}. If $f : A \rightarrow B$ is defined by $f(x) = \frac{1}{2}x + 1$ then represent f by (i) an arrow diagram (ii) a set of ordered pair (iii) a table (iv) a graph.

Solution:

$$f(x) = \frac{1}{2}x + 1$$

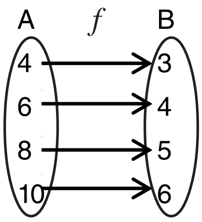
$$f(4) = \frac{1}{2} \times 4 + 1 = 2 + 1 = 3$$

$$f(6) = \frac{1}{2} \times 6 + 1 = 3 + 1 = 4$$

$$f(8) = \frac{1}{2} \times 8 + 1 = 4 + 1 = 5$$

$$f(10) = \frac{1}{2} \times 10 + 1 = 5 + 1 = 6$$

i) An arrow diagram



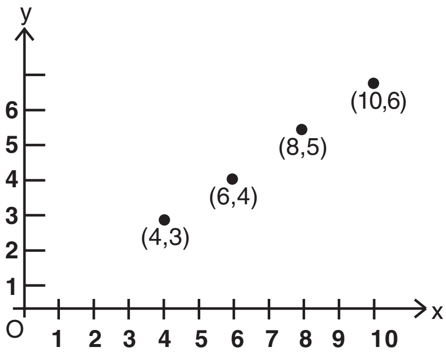
ii) Set of ordered pairs

f = {(4, 3) (6, 4) (8, 5) (10, 6)}

iii) Table

x	4	6	8	10
$f(x)$	3	4	5	6

iv) Graph



15. A function f : (-7, 6) $\rightarrow$ R is defined as following $f(x) = \begin{cases} x^2+2x+1 & -7 \le x < -5 \\ x+5 & -5 \le x \le 2 \\ x-1 & 2 < x < 6 \end{cases}$

find (i) 2f(-4) + 3f(2) ii) f(-7) -f(-3) iii) $\frac{4f(-3)+2f(4)}{f(-6)-3f(1)}$

$$f(x) = \begin{cases} x^2+2x+1 & -7 \le x < -5 & (-7,-6) \\ x+5 & -5 \le x \le 2 & (-5,-4,-3,-2,-1,0,1,2) \\ x-1 & 2 < x < 6 & (3,4,5) \end{cases}$$

i) 2f(-4) + 3f(2)

f(x) = x + 5

f(-4) = - 4 + 5 = 1

2xf(-4)= 1 x 2

2f(-4) = 2

f(2) = 2 + 5 = 7

3 x f(2) = 7 x 3

3f(2) = 21

2f(-4) + 3f(2) = 2 + 21

2f(-4) + 3f(2) = 23

ii) f(-7) - f(-3)

f(x) = x^2 + 2x + 1

f(-7) = $(-7)^2$ + 2x (- 7) + 1

= 49 - 14 + 1

= 50 - 14

f(-7) = 36

f(x) = x + 5

f(-3) = - 3 + 5

= 2

f(-3) = 2

f(-7) - f(-3) = 36 - 2

f(-7) - f (-3) = 34

iii) $\frac{4f(-3)+2f(4)}{f(-6)-3f(1)}$

f(x) = x + 5

f(-3) = - 3 + 5 = 2

4f(-3) = 2 x 4

4f(-3) = 8

f(x) = x - 1

f(4) = 4 - 1 = 3

2f(4) = 3 x 2

2f(4) = 6

4f(-3) + 2f(4) = 8 + 6

4f (-3) + 2f(4) = 14

f(x) = x^2 + 2x + 1

f(-6) = $(-6)^2$ + 2 x (- 6) + 1

= 36 - 12 + 1

= 37 - 12

f(-6) = 25

f(x) = x + 5

f(1) = 1 + 5 = 6

3f(1) = 6 x 3

3f(1) = 18

f(-6) - 3f(1) = 25 - 18

f(-6) - 3f(1) = 7

$$\frac{4f(-3)+2f(4)}{f(-6)-3f(1)} = \frac{14}{7}$$

Ans : 2

16. Let A = { 6, 9, 15, 18, 21 }; B = { 1, 2, 4, 5, 6 } and f : A $\rightarrow$ B be defined by f(x) = . Represent f by (i) an arrow diagram (ii) a set of ordered pairs (iii) a table (iv) a graph .

Solution:

$$f(x) = \frac{x-3}{3}$$

$$f(6) = \frac{6-3}{3} = \frac{3}{3} = 1$$

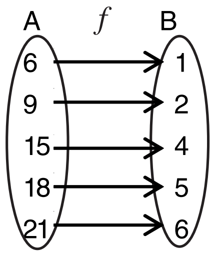
$$f(9) = \frac{9-3}{3} = \frac{6}{3} = 2$$

$$f(15) = \frac{15-3}{3} = \frac{12}{3} = 4$$

$$f(18) = \frac{18-3}{3} = \frac{15}{3} = 5$$

$$f(15) = \frac{21-3}{3} = \frac{18}{3} = 6$$

i) An arrow diagram



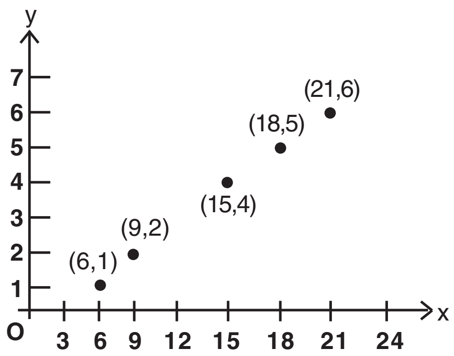
ii) a set of ordered pairs

f = {(6, 1), (9, 2), (15, 4), (18, 5), (21, 6)}

iii) a table

x	6	9	15	18	21
f(x)	1	2	4	5	6

iv) Graph



17. Let A = {5, 6, 7, 8}, B = {-11, - 4, 7, -1, -7, -9, -13} and f = {(x, y); y = 3-2x, x∈ A, y∈ B}

i) Write down the elements of f. ii) What is the co-domain iii) What is the range

iv) Identify the type of function.

Solution:

y = 3-2x

x = 5, y = 3-2 x 5 = 3 - 10 = -7

x = 6, y = 3 - 2 x 6 = 3 - 12 = -9

x = 7, y = 3 - 2 x 7 = 3 - 14 = -11

x = 8, y = 3 - 2 x 8 = 3 - 16 = -13

i) The elements of f

f = {(5, -7), (6, -9), (7, -11), (8, -13)}

ii) The co-domain = {-11, 4, 7, -10, -7, -9, -13}

iii) The range = {-7, -9, -11, -13}

iv) The type of function is

one - one function.

2. SEQUENCES AND SERIES OF REAL NUMBERS

1. The 10^{th} and 18^{th} terms of an A.P. are 41 and 73 respectively. Find the 27th term.

Solution : ★

Given that

$t_{10} = 41 \Rightarrow a + 9d = 41$ ---- (1)

$t_{18} = 73 \Rightarrow a + 17d = 73$ ---- (2)

(1) - (2) $\Rightarrow$ $-8d = 32$

$$d = \frac{-32}{-8}$$

d = 4 sub in (1) we get

$a + 9d = 41$

$a + 9 \times 4 = 41$

$a + 36 = 41$

$a = 5$

$a = 41 - 36$

$a = 5$

$t_{27} = a + 26\,d$

Sub a = 5 & d = 4, $t_{27} = 5 + 26\,(4)$

$= 5 + 104$

$t_{27} = 109$

2. If a, b, c are in A.P. then prove that $\frac{1}{bc}, \frac{1}{ca}, \frac{1}{ab}$ are also in A.P.

Solution:

If a, b, c are in A.P.

Divide each term by abc.

$\frac{a}{abc}, \frac{b}{abc}, \frac{c}{abc}$ are also in A.P.

$\frac{1}{bc}, \frac{1}{ca}, \frac{1}{ab}$ are also in A.P.

3. The 4th term of a geometric sequence of is $\frac{2}{3}$ and the seventh term is $\frac{16}{81}$. Find the geometric sequence.

$t_4 = \frac{2}{3} \Rightarrow ar^3 = \frac{2}{3}$ --- (1)

$t_7 = \frac{16}{81} \Rightarrow ar^6 = \frac{16}{81}$ --- (2)

$(2) \div (1) \Rightarrow \frac{ar^6}{ar^3} = \frac{16/81}{2/3}$

$r^{6-3} = \frac{16}{81} \times \frac{3}{2}$

$r^3 = \frac{8}{27}$

$r^3 = \left(\frac{2}{3}\right)^3$

$r = \frac{2}{3}$ sub in (1) we get

$ar^3 = \frac{2}{3}$

$a \times \left(\frac{2}{3}\right)^3 = 2/3$

$a = \frac{2}{3} \times \frac{3}{2} \times \frac{3}{2} \times \frac{3}{2}$

$a = \frac{9}{4}$

G.P. is a, ar, ar^2 .....

$\frac{9}{4}, \left(\frac{9}{4}\right)\left(\frac{2}{3}\right), \left(\frac{9}{4}\right)\left(\frac{2}{3}\right)^2 .....$

4. In a geometric sequence , the first term is $\frac{1}{3}$ and the sixth term is $\frac{1}{729}$, find the G.P.

$a = \frac{1}{3}$

$t_{6} = \frac{1}{729} \quad ar^5 = \frac{1}{729}$

$\left(\frac{1}{3}\right) r^5 = \frac{1}{729}$

$r^5 = \frac{1}{729} \times 3$

$= \frac{1}{243}$

$r^5 = \frac{1}{3^5}$

$r^5 = \left(\frac{1}{3}\right)^5$

$r = \frac{1}{3}$

G.P. is a, ar, ar^2 .....

$= \frac{1}{3}, \left(\frac{1}{3}\right) \frac{1}{3}, \frac{1}{3}\left(\frac{1}{3}\right)^2 ......$

$= \frac{1}{3}, \frac{1}{9}, \frac{1}{27} .....$

5. If the 4th and 7th terms of a G.P. are 54 and 1458 respectively, find the G.P. 54 and 1458 respectively, find the G.P.

$t_4 = 54 \Rightarrow ar^3 = 54$ ---- (1)

$t_7 = 1458 \Rightarrow ar^6 = 1458$ ---- (2)

$$(2) \div (1) \quad \frac{ar^6}{ar^3} = \frac{1458}{54}$$

$r^3 = 27$

$r^3 = (3)^3$

$r = 3$ sub in (1) we get

$ar^3 = 54$

$a(3)^3 = 54$

$$a = \frac{54}{3\times3\times3}$$

$a = 2$

G.P. is a, ar, ar^2

$= 2, (2)(3), (2)(3)^2 \ldots$

$= 2, 6, 18 \ldots$

6. Find the sum of all 3 digit natural numbers, which are divisible by 8.

Three digits natural numbers are 100, 101, ... 999.

Three digits natural numbers divisible by 8 are 104, 112, 120, ... 992.

a = 104, d = 8, l = 992

Step 1 :

$$n = \left(\frac{\ell - a}{d}\right) + 1$$

$$= \left(\frac{992-104}{8}\right) + 1$$

$$= \left(\frac{888}{8}\right) + 1$$

$= 111 + 1$

$n = 112$

```
     12                        125
8 | 100 + 4 = 104         8 | 999 -7 = 992
  |  8                      |  8
  |----                     |----
  |  20                     |  19
  |  16                     |  16
  -----                     -----
     4+4                       39
                               32
                               --
                                7
```

Step 2:

$$S_n = \frac{n}{2}[a + \ell]$$

$$S_{112} = \frac{112}{2}[104 + 992]$$

$= 56 \times 1096$

$S_{112} = 61376$

7. Find the sume of all 3 digit natural numbers, which are divisible by 9.

Three digit natural numbers are 100, 101, ... 999.

Three digit natural numbers divisible by 9 are 108, 117, ... 999

a = 108, d = 9, ℓ = 999

Step 1 :

```
     11                    111
8 | 100 + 8           8 | 999 - 0
  |  9                  |  9
  |----                 |----
  |  10                 |  9
  |   9                 |  9
  -----                 -----
     1+8                    9
                            9
                           --
                            0
```

$$n = \left(\frac{\ell - a}{d}\right) + 1$$

$$= \left(\frac{999-108}{9}\right) + 1$$

$$= \left(\frac{891}{9}\right) + 1$$

$= 99 + 1$

$n = 100$

Step 2:

$S_n = \frac{n}{2}[a + \ell]$

$S_{100} = \frac{100}{2}[108 + 999]$

$= 56 \times 1107$

$S_{100} = 55350$

8. Find the sum of all natural numbers between 300 and 500 which are divisible by 11.

The natural numbers between 300 and 500, which are divisible by 11 are 308, 319, ... 495.

$a = 308,\ d = 11,\ l = 495$

Step 1 :

$n = \left(\frac{\ell - a}{d}\right) + 1$

$= \left(\frac{495 - 308}{11}\right) + 1$

$= \left(\frac{187}{11}\right) + 1$

$= 17 + 1$

$n = 18$

27
11 | 300 + 8 = 308
22
80
77
3+8

45
11 | 499 - 4 = 495
44
59
55
4

Step 2:

$S_n = \frac{n}{2}[a + \ell]$

$S_{18} = \frac{18}{2}[308 + 495]$

$= 9 \times 803$

$S_{18} = 7227$

9. Find the sum of all numbers between 100 and 200 which are not divisible by 5.

Numbers which are divisible by 5 are 105, 110, ... 195, $a = 105,\ d = 5,\ \ell = 195$

Step 1 :

$n = \left(\frac{\ell - a}{d}\right) + 1$

$= \left(\frac{195 - 105}{5}\right) + 1$

$= \left(\frac{90}{5}\right) + 1$

$= 18 + 1$

$n = 19$

$S_n = \frac{n}{2}[a + \ell]$

$S_{19} = \frac{19}{2}[105 + 195]$

$= 19 \times 150$

$S_{19} = 2850$

Step 2 :

The sum of natural nos are 101 + 102 + ... 199

$$\sum n = \frac{n(n+1)}{2}$$

101 + 102 + ... + 199 = (1 + 2 + ... + 199) - (1 + 2 + ... + 100)

$$= \frac{199\times 200}{2} - \frac{100\times 101}{2}$$

= 19900 - 5050

= 14850

Step 3:

Sum of numbers which are not divisible = 14850 - 2850

= 12000

10. Find the sum of first n terms of the series 6 + 66+ 666 +...

S_n = 6 + 66 + 666 + ... to n times

= 6 (1 + 11 + 111 + ... to n times)

$= \frac{6}{9}$ (9 + 99 + 999 + ... to n times)

$= \frac{2}{3}$ [(10 - 1) + (100 - 1) + (1000 - 1) ..... to n times]

$= \frac{2}{3}$ **[**(10 + 100 + 1000 + ... to n times) - n**]**

Here a = 10, r = 10>1

$$= \frac{2}{3}\left[\frac{10(10^n-1)}{10-1} - n\right]$$

$$S_n = \frac{a(r^n-1)}{r-1} = \frac{10(10^n-1)}{10-1}$$

$$S_n = \frac{2}{3}\left[\frac{10(10^n-1)}{9} - n\right]$$

11. Find the sum of first n terms of the series 7 + 77 + 777 +...

S_n = 7 + 77 + 777 + ... to n times

= 7 (1 + 11 + 111 + ... to n times)

$= \frac{7}{9}$ (9 + 99 + 999 + ... to n times)

$= \frac{7}{9}$ [(10 - 1) + (100 - 1) + (1000 - 1) ..... to n times]

$= \frac{7}{9}$ **[**(10 + 100 + 1000 + ... to n times) - n**]**

Here a = 10, r = 10>1

$$= \frac{7}{9}\left[\frac{10(10^n-1)}{10-1} - n\right]$$

$$S_n = \frac{a(r^n-1)}{r-1} = \frac{10(10^n-1)}{10-1}$$

$$S_n = \frac{7}{9}\left[\frac{10(10^n-1)}{9} - n\right]$$

12. Find the sum of first n terms of the series 1 + 11 + 111 +... to 20 terms.

S_n = 1 + 11 + 111 + ... to n times

$= \frac{1}{9}$ (9 + 99 + 999 + ... to n times)

$= \frac{1}{9}$ [(10 - 1) + (100 - 1) + (1000 - 1) ..... to n times]

$= \frac{1}{9}$ **[**(10 + 100 + 1000 + ... to n times) - n**]**

$$S_n = \frac{1}{9}\left[\frac{10(10^n-1)}{10-1} - n\right] \qquad a = 10,\ r = 10>1$$

If r > 1

$$S_n = \frac{a(r^n-1)}{r-1}$$

$$S_{20} = \frac{1}{9}\left[\frac{10(10^{20}-1)}{10-1} - 20\right]$$

$$= \frac{1}{9}\left[\frac{10(10^{20}-1)}{9} - 20\right]$$

$$S_{20} = \left[\frac{10}{81}(10^{20}-1) - \frac{20}{9}\right]$$

13. Find the sum of the series $16^2 + 17^2 + 18^2 .... + 25^2$

$$\sum n^2 = \frac{n(n+1)(2n+1)}{6}$$

$16^2 + 17^2 + 18^2 .... + 25^2 = (1^2 + 2^2 + ... + 25^2) - (1^2 + 2^2 .... + 15^2)$

$$= \left(\frac{25\times26\times51}{6}\right) - \left(\frac{15\times16\times31}{6}\right)$$

= (25 x 13 x 17) - (5 x 8 x 31)
= 5525 - 1240
= 4285

14. Find the sum of series $16^2 + 17^2 + ... 35^2$

$$\sum n^2 = \frac{n(n+1)(2n+1)}{6}$$

$16^2 + 17^2 + ... + 35^2 = (1^2 + 2^2 + ... + 35^2) - (1^2 + 2^2 .... + 15^2)$

$$= \left(\frac{35\times36\times71}{6}\right) - \left(\frac{15\times16\times31}{6}\right)$$

= (35 x 6 x 17) - (5 x 8 x 31)
= 14910 - 1240
= 13670

15. Find the total area of 14 squares whose sides are 11 cm, 12 cm, ... 24 cm.

Area = $11^2 + 12^2 + 13^2 + ... + 24^2$

$$\sum n^2 = \frac{n(n+1)(2n+1)}{6}$$

$11^2 + 12^2 + 13^2 .... + 24^2 = (1^2 + 2^2 + ... + 24^2) - (1^2 + 2^2 .... + 10^2)$

$$= \left(\frac{24\times25\times49}{6}\right) - \left(\frac{10\times11\times21}{6}\right)$$

= (4 x 25 x 49) - (5 x 11 x 7)
= 4900 - 385
= 4515

Total Area = 4515 cm^2

16. Find the total area of 12 squares whose sides are 12 cm, 13cm, ... 23cm. respectively. **(June 12)** ★

Solution :

Given that the side length of 12 squares are 12cm, 13 cm, 14 cm ...... 23 cm.

Total area of the 12 squares is

Area = $12^2 + 13^2 + 14^2 + ... + 23^2$

$$\sum n^2 = \frac{n(n+1)(2n+1)}{6}$$

$12^2 + 13^2 + ... + 23^2 = (1^2 + 2^2 + ... + 23^2) - (1^2 + 2^2 + ... + 11^2)$

$$= \frac{23 \times 24 \times 47}{6} - \frac{11 \times 12 \times 23}{6}$$

= 23 x 4 x 47 - 22 x 23

= 4324 - 506

= 3818

Total Area = 3818 cm^2.

17. Find the total volume of 15 cubes whose edges are 16 cm, 17 cm, 18 cm, ..., 30 cm respectively.

Solution :

Given that the sides of the cubes are 16cm, 17cm, 18cm,.... 30 cm respectively.

Volume $= 16^3 + 17^3 + 18^3 + ... + 30^3$

$$\sum n^3 = \left[\frac{n(n+1)}{2}\right]^2$$

$$16^3 + 17^3 + 18^3 + ... + 30^3 = (1^3 + 2^3 + ... + 30^3) - (1^3 + 2^3 + ... + 15^3)$$

$$= \left(\frac{30 \times 31}{2}\right)^2 - \left(\frac{15 \times 16}{2}\right)^2$$

$$= (15 \times 31)^2 - (15 \times 8)^2$$

$$= (465)^2 - (120)^2$$

$$= (465 + 120)(465 - 120)$$

$$= 585 \times 345$$

Total Volume = 201825 cm^3

18. The sum of three consecutive terms in an A.P. is 6 and their product is -120. Find the three numbers.

Let three terms of an A.P. are a - d, a, a+d

Their sum = 6

a - d + a + a + d = 6

3a = 6

a = 6/3

a = 2

Their product = - 120

(a-d) (a) (a+d) = -120

$(a^2 - d^2)$ a = -120

sub a = 2

$(2^2 - d^2)2 = -120$

$$4 - d^2 = \frac{-120}{2}$$

$-d^2 = -60 - 4$

$d^2 = 64$

$d = \sqrt{8 \times 8}$

$d = \pm 8$

Three numbers are,

If a = 2; d = 8 $\Rightarrow$ 2- 8, 2, 2+8 = -6, 2 10 (or)

(or) If a = 2, d = -8 $\Rightarrow$ 2 - (-8), 2, 2 - 8 = 10, 2, -6

19. Find the sum of series 5 + 11 + 17 + ... + 95

a = 5, d = 11-5 = 6; ℓ = 95

Step 1:

$$n = \left(\frac{\ell - a}{d}\right) + 1$$

$$= \left(\frac{95 - 5}{6}\right) + 1$$

$$= \left(\frac{90}{6}\right) + 1$$

$$= 15 + 1$$

$$n = 16$$

Step 2:

$$S_n = \frac{n}{2}[a + \ell]$$

$$S_{16} = \frac{16}{2}[5+95]$$

$$= 7 \times 100$$

$$S_{16} = 800$$

3. ALGEBRA

1. Using elimination method, solve $101x + 99y = 499$, $99x + 101y = 501$

$101x + 99y = 499$ --- (1)

$99x + 101y = 501$ --- (2)

(1)+(2) $200x + 200y = 1000$

÷ 200

$x + y = 5$ --- (3)

(1)-(2) $2x - 2y = -2$

÷ 2

$x - y = -1$ ---- (4)

(3)+(4) $2x = 4$

$x = 4/2 = 2$

Substitute in (1)

$2 + y = 5$

$y = 5 - 2$

$y = 3$

$x = 2$

$y = 3$

2. Factorise : $x^3 - 2x^2 - 5x + 6$

1	1	-2	-5	6	
	0	1	-1	-6	
3	1	-1	-6	0	(x-1) is a factor
	0	3	6		
	1	2	0		(x-3) is a factor

(x +2) is a factor

(x - 1), (x - 3), (x+2) are factors.

3. Factorize $4x^3 - 7x + 3$

1	4	0	-7	3
	0	4	4	-3
	4	4	-3	0

$(x - 1)$ is a factor

$4x^2 + 4x - 3 = (2x + 3)(2x - 1)$

$\therefore$ $(x-1), (2x-1), (2x+3)$ are factors.

4. Factorize $x^3 - 7x + 6$

1	1	0	-7	6
	0	1	1	-6
2	1	1	-6	0
	0	2	6	
	1	3	0	

$(x-1)$ is a factor

$(x-2)$ is a factor

$(x+3)$ is a factor

$\therefore$ $(x-1), (x-2), (x+3)$ are factors.

5. Factorise: $x^3 - 3x^2 - 10x + 24$

2	1	-3	-10	24
	0	2	-2	-24
-3	1	-1	-12	0
	0	-3	12	
	1	-4	0	

$(x - 2)$ is a factor

$(x+3)$ is a factor

$(x-4)$ is a factor.

$(x-2)(x + 3)(x - 4)$ are factors.

(Note : If it is not possible to find all the factors leave as it is. In Ex. 3.5 problems IV, VIII and XI are all of the same type)

6. If $P = \frac{x}{x+y}$, $Q = \frac{y}{x+y}$, then find $\frac{1}{P-Q} - \frac{2Q}{P^2 - Q^2}$

$$\frac{1}{P-Q} - \frac{2Q}{P^2 - Q^2} = \frac{1}{P-Q} - \frac{2Q}{(P+Q)(P-Q)}$$

$$= \frac{P+Q-2Q}{(P+Q)(P-Q)}$$

$$= \frac{P-Q}{(P+Q)(P-Q)}$$

$$= \frac{1}{P+Q}$$

$$= \frac{1}{\frac{x}{x+y} + \frac{y}{x+y}}$$

$$= \frac{1}{\frac{x+y}{x+y}}$$

$$= 1$$

7. Find the square root of $(x^2 - 25)(x^2 + 8x + 15)(x^2 - 2x - 15)$

$$= (x + 5)(x - 5)(x + 3)(x + 5)(x - 5)(x + 3)$$

$$= (x+5)^2 (x - 5)^2 (x + 3)^2$$

Square root $= |(x + 5)(x - 5)(x + 3)|$

8. Find the square root of $9x^4 + 12x^3 + 10x^2 + 4x + 1$

	3	2	1		
3	9	12	10	4	1
	9				
6 2		12	10		
		12	4		
6 4 1			6	4	1
			6	4	1
				0	

Square root $= |3x^2 + 2x + 1|$

9. Find the square root of $x^4 - 10x^3 + 37x^2 - 60x + 36$

	1	-5	6		
1	1	-10	37	-60	36
	1				
2 -5		-10	37		
		-10	25		
2 -10 6			12	-60	36
			12	-60	36
				0	

Square root $= |x^2 - 5x + 6|$

10. Find the square root of $4+25x^2-12x-24x^3+16x^4$

Write in descending powers

$16x^4 - 24x^3 + 25x^2 - 12x + 4$

	4	– 3	2		
4	16	– 24	25	– 12	4
	16				
8 –3		–24	25		
		–24	9		
8 –6 2			16	– 12	4
			16	– 12	4
				0	

Square root $= |4x^2 - 3x + 2|$

11. If $m - nx + 28x^2 + 12x^3 + 9x^4$ is a perfect square, then find the values of m and n.

Write in descending order

$9x^4 + 12x^3 + 28x^2 - nx + m$

	3	2	4		
3	9	12	28	-n	m
	9				
6 2		12	28		
		12	4		
6 4 4			24	-n	m
			24	16	16
				0	

m = 16, n = -16

12. Find a and b if $ax^4 - bx^3 + 40x^2 + 24x + 36$ is a perfect square.

Write in ascending order

$36 + 24x + 40x^2 - bx^3 + ax^4$

	6	2	3		
6	36	24	40	-b	a
	36				
12 2		24	40		
		24	4		
12 4 3			36	-b	a
			36	12	9
				0	

a = 9

b = -12

13. The sum of a number and its reciprocal is $5\frac{1}{5}$, find the number.

Solution :

Let x be the number

$1/x$ be its reciprocal

$$\text{Sum} = 5\frac{1}{5}$$

$$x + \frac{1}{x} = \frac{26}{5}$$

$$\frac{x^2+1}{x} = \frac{26}{5}$$

$5(x^2+1) = 26x$

$5x^2 + 5 - 26x = 0$

$5x^2 - 26x + 5 = 0$

$(5x - 1)(x - 5) = 0$

$5x - 1 = 0$ or $x = 5$

$x = 1/5$ or $x = 5$

The number = {1/5, 5}

14. If the equation $(1+m^2)x^2 + 2mcx + c^2 - a^2 = 0$ has equal roots, then prove that $c^2 = a^2(1+m^2)$

Given equation $(1+m^2)x^2 + 2mcx + c^2 - a^2 = 0$

$a = 1 + m^2$, $b = 2mc$, $c = c^2 - a^2$

equal roots = $b^2 - 4AC = 0$

$(2mc)^2 - 4(1+m^2)(c^2 - a^2) = 0$

$4m^2c^2 - 4(c^2 - a^2 + m^2c^2 - m^2a^2) = 0$

$4m^2c^2 - 4c^2 + 4a^2 - 4m^2c^2 + 4m^2a^2 = 0$

$-4c^2 = -4a^2 - 4m^2a^2 = 0$

$\div -4$

$c^2 = a^2 + m^2a^2$

$c^2 = a^2(1+m^2)$

$\therefore c^2 = a^2(1+m^2)$

Thus proved.

4. MATRICES

1. Prove that $\begin{pmatrix} 3 & 5 \\ 1 & 2 \end{pmatrix}$ and $\begin{pmatrix} 2 & -5 \\ -1 & 3 \end{pmatrix}$ are multiplicative inverse to each other.

Solution:

$$\begin{pmatrix} 3 & 5 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 2 & -5 \\ -1 & 3 \end{pmatrix} = \begin{pmatrix} 6-5 & -15+15 \\ 2-2 & -5+6 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$$

Also $\begin{pmatrix} 2 & -5 \\ -1 & 3 \end{pmatrix}\begin{pmatrix} 3 & 5 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 6-5 & 10-10 \\ -3+3 & -5+6 \end{pmatrix}$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$$

The given matrices are inverses to each other under matrix multiplication.

2. Prove that $A = \begin{pmatrix} 5 & 2 \\ 7 & 3 \end{pmatrix}$ $B = \begin{pmatrix} 3 & -2 \\ -7 & 5 \end{pmatrix}$ are inverses to each other under matrix multiplication.

Solution :

$$AB = \begin{pmatrix} 5 & 2 \\ 7 & 3 \end{pmatrix}\begin{pmatrix} 3 & -2 \\ -7 & 5 \end{pmatrix}$$

$$= \begin{pmatrix} 15-14 & -10+10 \\ 21-21 & -14+15 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$$

$$BA = \begin{pmatrix} 3 & -2 \\ -7 & 5 \end{pmatrix}\begin{pmatrix} 5 & 2 \\ 7 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 15-14 & 6-6 \\ -35+35 & -14+15 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$$

The given matrices are inverses to each other under matrix multiplication.

3. If $A = \begin{pmatrix} 3 & 2 \\ 4 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 3 & 0 \\ 3 & 2 \end{pmatrix}$ then find AB and BA. Are they equal?

Solution:

$$AB = \begin{pmatrix} 3 & 2 \\ 4 & 0 \end{pmatrix}\begin{pmatrix} 3 & 0 \\ 3 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 9+6 & 0+4 \\ 12+0 & 0+0 \end{pmatrix} = \begin{pmatrix} 15 & 4 \\ 12 & 0 \end{pmatrix}$$

$$BA = \begin{pmatrix} 3 & 0 \\ 3 & 2 \end{pmatrix}\begin{pmatrix} 3 & 2 \\ 4 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 9+0 & 6+0 \\ 9+8 & 6+0 \end{pmatrix} =$$

$$BA = \begin{pmatrix} 9 & 6 \\ 18 & 6 \end{pmatrix}$$

$AB \neq BA$

4. If $A = \begin{pmatrix} -2 \\ 4 \\ 5 \end{pmatrix}$ and B = (1 3 -6) then verify that $(AB)^T = B^T A^T$.

Solution:

$$AB = \begin{pmatrix} -2 \\ 4 \\ 5 \end{pmatrix} (1 \quad 3 \quad -6)$$

$$= \begin{pmatrix} -2 & -6 & 12 \\ 4 & 12 & -24 \\ 5 & 15 & -30 \end{pmatrix}$$

$$(AB)^T = \begin{pmatrix} -2 & 4 & 5 \\ -6 & 12 & 15 \\ 12 & -24 & -30 \end{pmatrix} \quad \text{------ (1)}$$

$$B^T = \begin{pmatrix} 1 \\ 3 \\ -6 \end{pmatrix}$$

$$A^T = (-2 \quad 4 \quad 5)$$

$$B^T A^T = \begin{pmatrix} 1 \\ 3 \\ -6 \end{pmatrix} (-2 \quad 4 \quad 5)$$

$$= \begin{pmatrix} -2 & 4 & 5 \\ -6 & 12 & 15 \\ 12 & -24 & -30 \end{pmatrix} \quad \text{--- (2)}$$

From (1) and (2) we get $(AB)^T = B^T A^T$

5. If $A = \begin{pmatrix} 5 & 2 \\ 7 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix}$ verify that $(AB)^T = B^T A^T$.

Solution :

$$AB = \begin{pmatrix} 5 & 2 \\ 7 & 3 \end{pmatrix} \begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 10-2 & -5+2 \\ 14-3 & -7+3 \end{pmatrix}$$

$$AB = \begin{pmatrix} 8 & -3 \\ 11 & -4 \end{pmatrix}$$

$$(AB)^T = \begin{pmatrix} 8 & 11 \\ -3 & -4 \end{pmatrix} \quad \text{------ (1)}$$

$$B^T = \begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix}$$

$$A^T = \begin{pmatrix} 5 & 7 \\ 2 & 3 \end{pmatrix}$$

$$B^TA^T = \begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} 5 & 7 \\ 2 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 10-2 & 14-3 \\ -5+2 & -7+3 \end{pmatrix}$$

$$B^TA^T = \begin{pmatrix} 8 & 11 \\ -3 & -4 \end{pmatrix} \quad \text{----- (2)}$$

From (1) and (2), we get

$$(AB)^T = B^T A^T.$$

6. If $A = \begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix}$ then show that $A^2 - 4A + 5I_2 = 0$

Solution

$$A^2 = A \times A = \begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 1-2 & -1-3 \\ 2+6 & -2+9 \end{pmatrix}$$

$$= \begin{pmatrix} -1 & -4 \\ 8 & 7 \end{pmatrix}$$

$$4A = 4\begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 4 & -4 \\ 8 & 12 \end{pmatrix}$$

$$5I_2 = 5 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix}$$

$$A^2 - 4A = 5I_2 = \begin{pmatrix} -1 & -4 \\ 8 & 7 \end{pmatrix} - \begin{pmatrix} 4 & -4 \\ 8 & 12 \end{pmatrix} + \begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix}$$

$$= \begin{pmatrix} -1-4+5 & -4+4+0 \\ 8-8+0 & 7-12+5 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$A^2 - 4A + 5I_2 = 0$$

7. If $A = \begin{pmatrix} 3 & 2 \\ -1 & 4 \end{pmatrix}$, $B = \begin{pmatrix} -2 & 5 \\ 6 & 7 \end{pmatrix}$ and $C = \begin{pmatrix} 1 & 1 \\ -5 & 3 \end{pmatrix}$. Verify that A(B+C) = AB + AC.

Solution:

$$B + C = \begin{pmatrix} -2 & 5 \\ 6 & 7 \end{pmatrix} + \begin{pmatrix} 1 & 1 \\ -5 & 3 \end{pmatrix} = \begin{pmatrix} -1 & 6 \\ 1 & 10 \end{pmatrix}$$

$$A(B+C) = \begin{pmatrix} 3 & 2 \\ -1 & 4 \end{pmatrix} \begin{pmatrix} -1 & 6 \\ 1 & 10 \end{pmatrix}$$

$$= \begin{pmatrix} -3+2 & 18+20 \\ 1+4 & -6+40 \end{pmatrix}$$

$$= \begin{pmatrix} -1 & 38 \\ 5 & 34 \end{pmatrix} \qquad \text{----- (1)}$$

$$AB = \begin{pmatrix} 3 & 2 \\ -1 & 4 \end{pmatrix} \begin{pmatrix} -2 & 5 \\ 6 & 7 \end{pmatrix}$$

$$= \begin{pmatrix} -6+12 & 15+14 \\ 2+24 & -5+28 \end{pmatrix}$$

$$= \begin{pmatrix} 6 & 29 \\ 26 & 23 \end{pmatrix}$$

$$AC = \begin{pmatrix} 3 & 2 \\ -1 & 4 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ -5 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 3-10 & 3+6 \\ -1-20 & -1+12 \end{pmatrix}$$

$$= \begin{pmatrix} -7 & 9 \\ -21 & 11 \end{pmatrix}$$

$$AB + AC = \begin{pmatrix} 6 & 29 \\ 26 & 23 \end{pmatrix} + \begin{pmatrix} -7 & 9 \\ -21 & 11 \end{pmatrix}$$

$$= \begin{pmatrix} -1 & 38 \\ 5 & 34 \end{pmatrix} \qquad \text{----- (2)}$$

From (1) and (2) we get

A (B+C) = AB + AC

5. COORDINATE GEOMETRY

1. Find the points of trisection of the line segment joining the points A(2,-2) and B(-7, 4).

Let P and Q be the points of trisection of AB such that AP = PQ = QB. Then the point P divides AB internally in the ratio 1 : 2 and Q divides AB internally in the ratio 2 : 1

$$P = \left(\frac{1\times(-7)+(2\times2)}{1+2}, \frac{(1\times4)+2(-2)}{1+2} \right)$$

$$= \left(\frac{-7+4}{3}, \frac{4-4}{3} \right) = (-1, 0)$$

Thus, the point P is (-1, 0)
Again by section formula

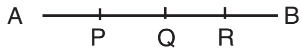
$$Q = \left(\frac{2\times(-7)+(1\times2)}{2+1}, \frac{(2\times4)+1\times(-2)}{2+1}\right)$$

$= (-4, 2)$

Thus the point Q is (-4, 2)

2. Find the points which divide the line segment joining A (-4, 0) and (0,6) into four equal parts.

Let P,Q,R be the points which divide AB into four equal parts.



The point Q is the midpoint of AB

$$Q = \left(\frac{-4+0}{2}, \frac{0+6}{2}\right) = (-2, 3)$$

now P is the midpoint of AQ.

$$P = \left(\frac{-4-2}{2}, \frac{0+3}{2}\right)$$

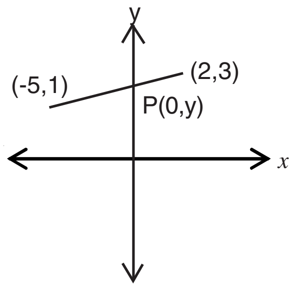
$$P = \left(\frac{-6}{2}, \frac{3}{2}\right) = \left(-3, \frac{3}{2}\right)$$

R is the midpoint of QB.

$$R = \left(\frac{-2+0}{2}, \frac{3+6}{2}\right) = \left(-1, \frac{9}{2}\right)$$

Hence the required points are $P = \left(-3, \frac{3}{2}\right)$ Q = (-2, 3) $R = \left(-1, \frac{9}{2}\right)$.

3. In what ratio is the line joining the points (-5, 1) and (2, 3) divided by the y-axis. Also find the point of intersection.



Let A (-5, 1) and B(2, 3) be the given points.

Let P (0, y) divide AB internally in the ratio ℓ: m. By section formula.

$$P(0, y) = P\left(\frac{(\ell\times2)+(m\times(-5)}{\ell+m}, \frac{(\ell\times3)+m\times1)}{\ell+m}\right) \quad \text{---- (1)}$$

$$P(0, y) = P\left(\frac{2\ell-5m}{\ell+m}, \frac{3\ell+m}{\ell+m}\right) \quad \text{---- (1)}$$

Equating x coordinates to zero.

$$\frac{2\ell-5m}{\ell+m} = 0 \Rightarrow 2l - 5m = 0 \Rightarrow \frac{\ell}{m} = \frac{5}{2}$$

The required ratio is 5 : 2

Also from (1) we have $P(0,y) = P\left(0, \frac{(5\times3)+(2\times1)}{5+2}\right)$

$$= P\left(0, \frac{17}{7}\right)$$

Hence the required point of intersection is $\left(0, \frac{17}{7}\right)$

ℓ : m = 5 : 2 and P(0, y) = P (0, 17/4)

4. Find the length of the medians of the triangle whose vertices are (1, -1) , (0, 4) and (-5, 3).

Solution :

Let A (1, -1), B (0, 4) C (-5, 3) be the vertices of the triangle.

Let D, E, F be the mid points of BC, AC and AB.

The midpoint of BC is D $= \left(\frac{0-5}{2}, \frac{4+3}{2}\right) = D\left(\frac{-5}{2}, \frac{7}{2}\right)$

The midpoint of AC is E $= \left(\frac{1-5}{2}, \frac{-1+3}{2}\right) = E(-2, 1)$

The midpoint of AB is F $= \left(\frac{1+0}{2}, \frac{-1+4}{2}\right) = F\left(\frac{1}{2}, \frac{3}{2}\right)$

The length of the median AD $= \sqrt{\left(1+\frac{5}{2}\right)^2 + \left(-1-\frac{7}{2}\right)^2}$

$$= \sqrt{\left(\frac{7}{2}\right)^2 + \left(\frac{-9}{2}\right)^2} = \sqrt{\frac{49}{4}+\frac{81}{4}} = \sqrt{\frac{130}{4}}$$

The length of the median BE $= \sqrt{(2-0)^2 + (1-4)^2} = \sqrt{4+9} = = \sqrt{13}$

The length of the median CF $= \sqrt{\left(\frac{1}{2}+5\right)^2 + \left(\frac{3}{2}-3\right)^2} = \sqrt{\left(\frac{11}{2}\right)^2 + \left(\frac{-3}{2}\right)^2} = \sqrt{\frac{121}{4}+\frac{9}{4}} = \sqrt{\frac{130}{4}}$

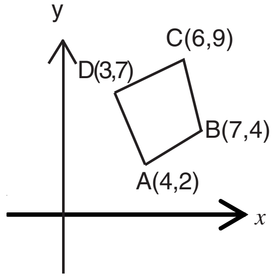
Thus the length of medians of the ΔABC are $\frac{\sqrt{130}}{2}$, $\sqrt{13}$, $\frac{\sqrt{130}}{2}$

5. Find the area of the quadrilateral whose vertices are (6, 9), (7, 4), (4,2) and (3,7).

Plot the given points in a rough diagram and take the vertices in counter clockwise direction.

Let the given points be A(4,2), B(7,4), C (6, 9), D(3,7).

Area of the quadrilateral ABCD.



$$= \frac{1}{2}\begin{Bmatrix} 4 & 7 & 6 & 3 & 4 \\ 2 & 4 & 9 & 7 & 2 \end{Bmatrix}$$

$$= \frac{1}{2}[(16+63+42+6)-(14+24+27+28)]$$

$$= \frac{1}{2}[127-93]$$

$$= \frac{1}{2}\times 34$$

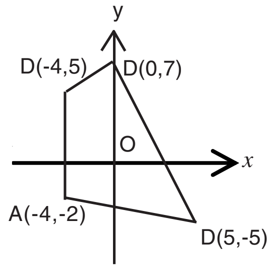
= 17

Thus the area of the quadrilateral ABCD is 17 sq. units.

6. Find the area of the quadrilateral whose vertices are (-4, 5) (0,7) (5,-5) and (-4,-2).

Plot the given points in a rought diagram and take the vertices in counter clockwise direction.

Area of the quadrilateral ABCD



$$= \frac{1}{2}\begin{Bmatrix} -4 & 5 & 0 & -4 & -4 \\ -2 & -5 & 7 & 5 & -2 \end{Bmatrix}$$

$$= \frac{1}{2}[(20+35+0+8) - (-10 + 0 - 28 - 20)]$$

$$= \frac{1}{2}[63 + 58]$$

$$= \frac{1}{2} \times 121$$

= 60.5

Thus, the area of the quadrilateral ABCD is 605 sq.units.

7. Find the value of k for which the given points are collinear (2, -5) (3, -4) and (9, k).

Let the given points be A(2, -5) B(3, -4) C (9, k).

The given points are collinear.

Thus area of $\Delta ABC = 0$

$$D = \frac{1}{2}\begin{bmatrix} 2 & 3 & 9 & 2 \\ -5 & -4 & k & 5 \end{bmatrix} = 0$$

$$D = \frac{1}{2}[(-8 + 3k - 45) - (-15 - 36 + 2k)] = 0$$

$$D = \frac{1}{2}[-53 + 3k + 51 - 2k] = 0$$

D = -2 + k = 0

k - 2 = 0

k = 2

Thus the value of k is 2.

8. Find the area of the triangle formed by joining the midpoints of the sides of a triangle whose vertices are (0,- 1), (2,1) and (0,3). Find the ratio of this area to the area of the given triangle.

Vertices are (0,- 1), 2,1),(0,3) find the ratio of this area to the area of the given triangle.

Let the vertices of the triangle be a(0,-1) B(2,1) and C (0,3).

Let D, E, F be the mid points of the sides BC, CA and AB respectively.

D is the mid point of BC

The mid point of BC is D $= \left(\frac{2+0}{2}, \frac{1+3}{2}\right)$ = D (1, 2)

The mid point of AC is E $= \left(\frac{0+0}{2}, \frac{3-1}{2}\right)$ = E (0, 1)

The mid point of AB is F $= \left(\frac{0+2}{2}, \frac{-1+1}{2}\right)$ = F (1, 0)

Thus Area of ΔDEF $= \frac{1}{2}\begin{Bmatrix} 1 & 0 & 1 & 1 \\ 2 & 1 & 0 & 2 \end{Bmatrix}$

$$= \frac{1}{2}[(1+0+2)-(0+1+0)] = 1 \text{ sq. unit}$$

Thus area of ΔDEF is 1 sq. units

Area of ΔABC $= \frac{1}{2}\begin{Bmatrix} 2 & 0 & 0 & 2 \\ 1 & 3 & -1 & 1 \end{Bmatrix}$

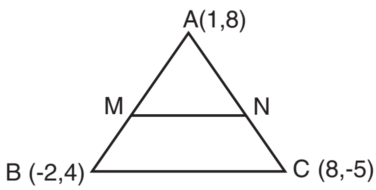
$$= \frac{1}{2}[(6+0+0)-(0+0-2)]$$

= 4 sq. units

Thus the area of the ΔABC is 4 sq.units. Hence, the ratio of the area of ΔDEF to the area of ΔABC is 1 : 4

9. The vertices of ΔABC are A (1, 8), B (-2, 4), C (8, -5). If M and N are the mid points of AB and AC respectively, find the slope of MN and hence verify that MN is parallel to BC.

Mid point M of AB is



$$M = \left(\frac{1-2}{2}, \frac{8+4}{2}\right)$$

$$= \left(\frac{-1}{2}, 6\right)$$

Mid point N of AC is

$$N = \left(\frac{1+8}{2}, \frac{-5+8}{2}\right)$$

$$N = \left(\frac{9}{2}, \frac{3}{2}\right)$$

The slope of the line MN is $M_1 = \frac{\frac{3}{2}-6}{\frac{9}{2}+\frac{1}{2}} = \frac{\frac{3-12}{2}}{\frac{10}{2}} = \frac{-9}{10}$

Also the slope of BC is $M_2 = \frac{-5-4}{8+2} = \frac{-9}{10}$

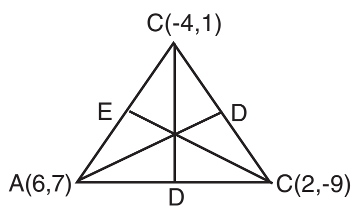
We have $M_1 = M_2$

Hence, the straight lines BC and MN are parallel.

10. A triangle has vertices at (6 , 7), (2 , -9) and (-4 , 1). Find the slopes of its medians.

Let the vertices be A(6 , 7), B(2 , -9) C(-4 , 1)

Let D, E,F be the midpoints of BC, CA, AB resp. then AD, BE and CF are the medians of the ΔABC.



The Mid point of BC is D = $\left(\frac{2-4}{2}, \frac{-9+1}{2}\right)$

= (-1, -4)

The midpoint of CA is E $\left(\frac{-4+6}{2}, \frac{1+7}{2}\right)$

E = (1, 4)

The midpoint of AB is F $\left(\frac{6+2}{2}, \frac{7-9}{2}\right)$

F = (4, -1)

Slope of AD $= \frac{-4-7}{-1-6} = \frac{-11}{-7} = \frac{11}{7}$

Slope of BE $= \frac{4+9}{1-2} = \frac{13}{-1} = -13$

Slope of CF $= \frac{-1-1}{4+4} = \frac{-2}{8} = \frac{-1}{4}$

Slopes of the medians are $\frac{11}{7}$, -13 and $\frac{-1}{4}$.

11. The line joining the points A (-2, 3) and B (a, 5) is parallel to the line joining the points C (0, 5) and D (-2, 1). Find the value of 'a'.

Since the lines AB and CD are parallel their slopes are equal.

Thus slope of AB = The slope of CD.

Slope of AB $= \frac{5-3}{a+2} = \frac{2}{a+2}$

Slope of CD $= \frac{1-5}{-2-0} = \frac{-4}{-2} = 2$

Equating the above

$\frac{2}{a+2} = 2$

a + 2 = 1

a = 1 - 2

a = -1

Hence, the value of a = -1

12. Find the equation of the straight line passing through the point (2, 2) and the sum of the intercepts is 9.

Solution:

Let x and y intercepts of the straight line be a and b respectively.

Then

a + b = 9 or b = 9 - a

Now, the equation of the straight line in intercepts from is $\frac{x}{a}+\frac{y}{b} = 1$ ---- I

Since I passes through (2, 2) we have $\frac{2}{a}+\frac{2}{9-a} = 1$

$\Rightarrow a^2 - 9a + 18 = 0$

$\Rightarrow (a - 6)(a - 3) = 0$

Thus a = 6 or a = 3

when a = 3 from the equation I we have $\frac{x}{3}+\frac{y}{6} = 1 \Rightarrow 2x + y - 6 = 0$

when a = 6 from the equation I we have $\frac{x}{6}+\frac{y}{3} = 1 \Rightarrow x + 2y - 6 = 0$

13. Find the equation of the line whose gradient is $\frac{3}{2}$ and which passes through P, where P divides the line segment joining A (-2, 6) and B (3, -4) in the ratio 2 : 3 internally.

The point P divides AB in the ratio 2 : 3 internaly.

Thus the point P is $\left(\frac{2(3)+3(-2)}{2+3}, \frac{2(-4)+3(6)}{2+3}\right)$

$= (2, 0)$

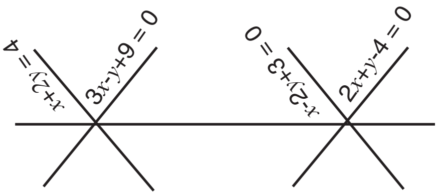
Hence equation of the straight line passing through (0, 2) with slope $\frac{3}{2}$ is

$$y - 2 = \frac{3}{2}(x - 0)$$

$$2y - 4 = 3x$$

$$3x - 2y + 4 = 0$$

14. Find the equation of the straight line joining the point of inter section of the lines 3x - y + 9 = 0 and $x + 2y = 4$ and the point of intersection of the lines $2x + y - 4 = 0$ and $x - 2y + 3 = 0$



Given equations can be written as

$3x - y = -9$ ----- (I) $x + 2y = 4$ ---- (II)

$2x + y = 4$ ---- (III) $x - 2y = -3$ --- (IV)

Solving I and II

$3x - y = -9$

$x + 2y = 4$

(I x 2) + II

$6x - 2y = -18$

$x - 2y = -4$

$7x = -14$

$x = -2$

Put x = -2 we get

$-2 + 2y = 4$

$2y = 4 + 2$

$y = 6/2 \quad y = 3$

Point of intersection is (-2, 3)

Solving III & IV $2x + y = 4$ ---- (III)

$x - 2y = -3$ ---- (IV)

(III x 2) + IV $4x + 2y = 8$

$x - 2y = -3$

$5x = 5 \quad x = 1$

Put x = 1

$1 - 2y = -3$

$-2y = -4$

$y = 2$

The point of intersection is (1, 2)

The equation of the straight line joining (-2, 3) and (1, 2) is

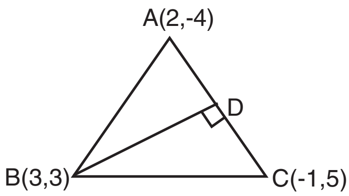
$$\frac{y-3}{2-3} = \frac{x+2}{1+2} \Rightarrow \frac{y-3}{-1} = \frac{x+2}{3}$$

$$\Rightarrow x + 3y - 7 = 0$$

$$M = \left(\frac{3-5}{2}, \frac{-2+8}{2}\right)$$

$= (-1, 3)$

15. If the vertices of a ΔABC are A(2, -4), B(3, 3) and C(-1, 5), then find the equation of the straight line along the altitude from the vertex B.



Let BD be the altitude from the vertex B

Now the slope of AC is $\frac{5+4}{-1-2} = \frac{9}{-3} = -3$

Thus, the slope of the straight line along te altitude BD is $\frac{1}{3}$ (AC $\perp$ BD)

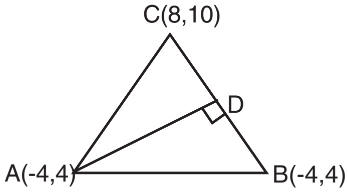
Now, the required line is passing through (3, 3) with slope $\frac{1}{3}$.

The required equation is $y - 3 = \frac{1}{3}(x - 3)$

$$3y - 9 = x - 3 \Rightarrow x - 3y + 6 = 0$$

16. If the vertices of a ΔABC are A (-4, 4) B(8,4) C (8,10) then find the equation of the straight line along the median from the vertex A.

Let AD be the median through the vertex.



Midpoint D of BC is D $\left(\frac{8+8}{2}, \frac{4+10}{2}\right)$

D = (8, 7)

The equation of the median AD joining A (-4, 4) D (8,7) is

$$\frac{y-4}{7-4} = \frac{x+4}{8+4} \Rightarrow 4y - 16 = x + 4$$

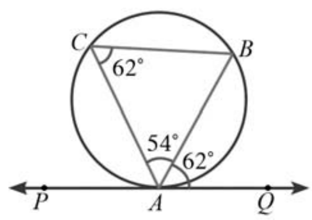
$$\Rightarrow x - 4y + 20 = 0$$

6. GEOMETRY

(For those who want to score more than 50% marks)

1. Let PQ be a tangent to a circle at A and AB be a chord. Let C be a point on the circle such that $\angle$BAC = 54° and $\angle$BAQ = 62°. Find $\angle$ABC.

PQ is a tangent. AB is a chord.



$\angle$BAQ = $\angle$ACB = 62° (Theorem)

$\angle$BAC = $\angle$ABC + $\angle$ACB = 180°

(Sum of three angles of a triangle)

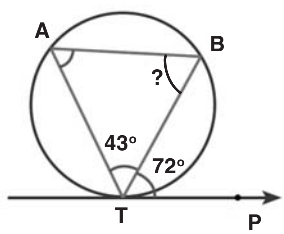
54 + $\angle$ABC+62° = 180°

$\angle$ABC +- 116 = 180

$\angle$ABC = 180 - 116 = 64°

$\angle$ABC = 64°

2. In the figure TP is a tangent to a circle. A and B are two points on the circle. If $\angle$BTP = 72° and $\angle$ATB = 43° find $\angle$ABT. (Ap. 13)



TP is a tangent.

TB is a chord.

$\angle$BTP = $\angle$BAT = 72° (Theorem)

$\angle$BTP + $\angle$ABT + $\angle$BAT = 180°

(Sum of three angles of a triangle)

43° + $\angle$ABT + 72° = 180°

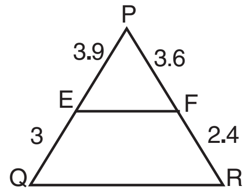
$\angle ABT + 115 = 180$

$\angle ABT = 180 - 115$

$= 65^\circ$

$\angle ABT = 65^\circ$

3. E and F are points on the sides PQ and PR respectively, of a ΔPQR. Verify EF||QR, If PE = 3.9 cm, EQ = 3 cm, PF = 3.6 cm and FR = 2.4 cm.



$$\frac{PE}{EQ} = \frac{PF}{FR}$$

$$\frac{3.9}{3} = \frac{3.6}{2.4}$$

$3.9 \times 2.4 = 3 \times 3.6$

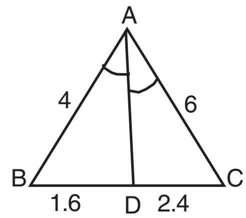
$9.36 \neq 10.8$

Hence EF $\nparallel$ QR (not prallel)

4. E and F are points on the sides PQ and PR respectively, of a ΔPQR. Verify EF || QR. If PE = 4 cm, QE = 4.5 cm, PF = 8 cm and RF = 9 cm.

(Try your self)

5. Check whether AD is the bisector of $\angle A$ of ΔABC. Where AB = 4 cm, AC = 6 cm, BD = 1.6 cm, and CD = 2.4 cm.



$$\frac{BD}{DC} = \frac{AB}{AC}$$

$$\frac{1.6}{2.4} = \frac{4}{6}$$

$1.6 \times 6 = 2.4 \times 4$

$9.6 = 9.6$

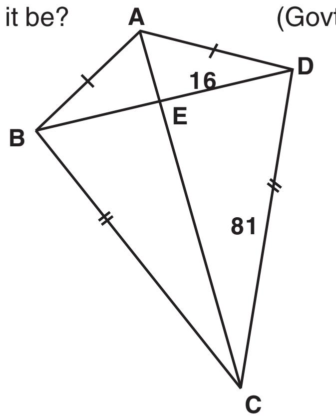
Hence AD is the internal bisector of $\angle A$.

6. Check whether AD is the bisector of $\angle A$ of ΔABC where AB = 6 cm, AC = 8 cm, BD = 1.5 cm and CD = 3 cm. (Try yourself)

Theorems to be learnt.

1. Basic proportionality theorem (or) Thales Theorem (Oct.14, Ap. 14, Ju. 13)
2. Angle Bisector Theorem (Oct. 13, Ap. 12)
3. Phythagoras Theorem. (Ap. 13, Ju. 12)

2. A boy is designing a diamond shaped kite, as shown in the figure where AE = 16cm, EC = 81cm. He wants to use a straight cross bar BD. How long should it be? (Govt. Model Question)



$\Delta EAD \sim \Delta EDC$

$$\frac{EA}{ED} = \frac{ED}{EC}$$

$ED^2 = EA \times EC$

$ED^2 = 16 \times 81$

$ED = \sqrt{16 \times 81}$

$= 4 \times 9$

$ED = 36$

ED = BE = 36 cm

BD = 36 + 36 = 72

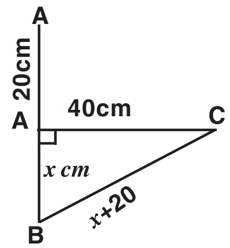
BD = 72 cm

length of the bar = 72 cm

3. A lotus is 20 cm above the water surface in a pond and its stem is partly below the water surface. As the wind blew, the stem is pushed aside so that the lotus touched the water 40 cm away from the original position of the stem. How much of the stem was below the water surface originally?

Solution:

BD = length below the water = x m



In ΔBCD, by phthagorus theorem,

$(hy)^2 = (si)^2 + (si)^2$

$(x + 20)^2 = x^2 + 40^2$

$x^2 + 40x + 400 = x^2 + 1600$

$x^2 + 40x + 400 = x^2 + 1600$

$40x = 1600 - 400$

$40 = 1200$

$x = \frac{1200}{40} = 30$

$x = 30$ cm

height of the stem below water = 30 cm

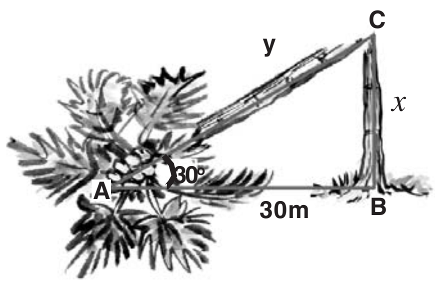
4. A point O in the interior of a rectangle ABCD is joined to each of the vertices A, B, C and D. Prove that $OA^2 + OC^2 + OB^2 + OD^2$ (Ju. 14)
5. If all sides of a parallelogram touch a circle, show that the parallelogram is a rhombus. (Ap. 15)
6. ABCD is a quadrilateral such that all of its sides touch a circle. If AB = 6 cm, BC = 6.5 cm and CD = 7 cm , then find the length of AD.
7. The image of a tree on the film of a camera is of length 35 mm, the distance from the lens to the film is 42 mm and the distance from the lens to the tree is 6 m. How tall is the portion of the tree being photographed? (Score Model Question)

7. TRIGONOMETRY

1. A vertical tree is broken by the wind. The top of the tree touches the ground and makes an angle 30° with it. If the top of the tree touches the ground 30 m away from its foot, then find the actual height of the tree.

Let the height of the tree be (x+y)m (Oct-12, July-14)

In ΔABC, $\tan\theta = \frac{\text{Opposite side}}{\text{adjacent side}}$



$\tan 30^\circ = \frac{x}{30}$

$\frac{1}{\sqrt{3}} = \frac{x}{30}$

$\sqrt{3}\, x = 1 \times 30$

$x = \frac{30}{\sqrt{3}}$

$$= \frac{30\sqrt{3}}{\sqrt{3}.\sqrt{3}} = \frac{30\sqrt{3}}{3} = 10\sqrt{3} \text{ m}$$

$x = 10\sqrt{3}$ m

$$\text{Cos}\theta = \frac{\text{adjacent side}}{\text{hypotenuse}}$$

$$\cos 30^\circ = \frac{30}{y}$$

$$\frac{\sqrt{3}}{2} = \frac{30}{y}$$

$\sqrt{3}\, y = 2 \times 30$

$$y = \frac{2\times30\times\sqrt{3}}{\sqrt{3}.\sqrt{3}} = \frac{2\times30\times\sqrt{3}}{3} = 20\sqrt{3}$$

$y = 20\sqrt{3}$ m

Height of the tree $= x + y$

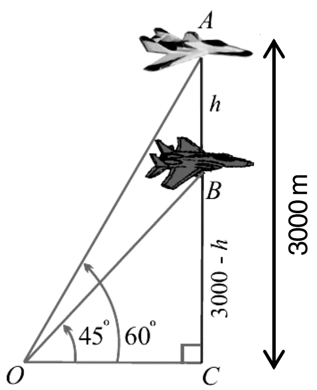
$= 10\sqrt{3} + 20\sqrt{3}$ m

Height of the tree $= 30\sqrt{3}$ m

2. A jet fighter at a height of 3000 m from the ground, passes directly over another jet fighter at an instance when their angles of elevation from the same observation point are 60° and 45° respectively. Find the distance of the first jet fighter from the second jet at that instant. ($\sqrt{3}$ =1.732) (Ju. 13)

Let the distance be h metre

In ΔOAC,



$$\tan\theta = \frac{\text{Opposite side}}{\text{adjacent side}}$$

$$\tan 60^\circ = \frac{3000}{OC}$$

$$\sqrt{3} = \frac{3000}{OC}$$

$\sqrt{3}\, OC = 3000$

$$OC = \frac{3000}{\sqrt{3}}$$

$$= \frac{3000\times\sqrt{3}}{\sqrt{3}\times\sqrt{3}} = \frac{3000\sqrt{3}}{3} = 1000\sqrt{3}$$

$OC = 1000\sqrt{3}$ m

In ΔOCB, $\tan\theta = \dfrac{\text{Opposite side}}{\text{adjacent side}}$

$$\tan 45^\circ = \frac{3000 - h}{OC}$$

$$1 = \frac{3000 - h}{1000\sqrt{3}}$$

$1 \times 1000\sqrt{3} = 3000 - h$

$h = 3000 - 1000\sqrt{3}$

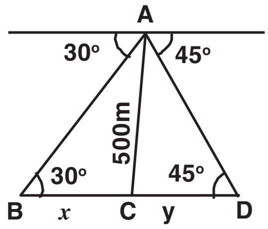
$h = 3000 - 1000 \times 1.732$

$h = 3000 - 1732$

$h = 1268$ m

distance between them = 1268m.

3. A person in an helicopter flying at a height of 500 m, observes two objects lying opposite to each other on either bank of a river. The angles of depression of the objects are 30° and 45°. Find the width of the river. ($\sqrt{3} = 1.732$) (Ap. 14)



Let the width of the river be = $x_m + y_m$

In ΔABC,

$\tan\theta = \frac{\text{Opposite side}}{\text{adjacent side}}$

$\tan 30° = \frac{500}{x}$

$\frac{1}{\sqrt{3}} = \frac{500}{x}$

$x = 500\sqrt{3}$

$x = 500 \times 1.732$

$= 866.00$

$x = 866$ m

In ΔACD, $\tan\theta = \frac{\text{Opposite side}}{\text{adjacent side}}$

$\tan 45 = \frac{500}{y}$

$1 = \frac{500}{y}$

$1 \times y = 500$

$y = 500$ m

Width of the river = $x + y$

$= 866 + 500$

$= 1366$ m

4. A person in an helicopter flying at a height of 700 m, observes two objects lying opposite to each other on either bank of a river. The angles of depression of the objects are 30° and 45°. Find the width of the river. ($\sqrt{3} = 1.732$)

Try this sum using the steps given in the previous sum. (Q.No. 3)

Those who want to score more than 60% may try and learn the following questions.

1. Page 200, Example 7.6 (Ap.13)
2. Page 203, Example 7.12 (Oct. 13, Ap. 15)
3. Page 204, Exercise 7.1, (5) (Score model question III)
4. Page 210, Example 7.20 (Oct. 14)
5. Page 211, Example 7.22 (Score model question V)
6. Page 216, Exercise 7.2 (10) (Ap. 13)

7. Page 216, Exercise 7.2 (12) (Ap. 12)
8. Page 216, Exercise 7.2 (17) (Ju. 12)
9. Page 216, Exercise 7.2 (16) (Score model question IV)

8. MENSURATION

1. The diameter of a road roller of length 120 cm is 84 cm. If it takes 500 complete revolutions to level a playground, then find the cost of levelling it at the cost of 75 paise per square metre. (Take $\pi = \frac{22}{7}$)

Solution:

Given that 2r = 84 m $\Rightarrow$ r = 42 cm, h = 120 cm

Area covered by the roller in one revolution = $2\pi rh$

$$= 2 \times \frac{22}{7} \times 42 \times 120$$

$$= 31680 \text{ cm}^2$$

Area covered by the roller in 500 revolutions = 31680 x 500

$$= 15840000 \text{ cm}^2$$

$$= \frac{15840000}{10000} \text{ m}^2$$

$$= 1584 \text{ m}^2$$

Cost of levelling per 1 sq. m = 75 paise

Thus, cost of levelling the play ground = Rs. 1584 x 0.75

= Rs. 1188

2. The total surface area of a solid right circular cylinder is 660 sq.cm. If its diameter of the base is 14 cm, find the height and curved surface area of the cylinder.

Solution:

Given that 2r = 14 cm $\Rightarrow$ r = 7 cm

Total surface area = 660sq.cm

$$2\pi r (h+r) = 660$$

$$2 \times \frac{22}{7} \times 7 (h + 7) = 660$$

$$h = \frac{660}{2 \times 22} - 7$$

$$= 15 - 7$$

$$= 8 \text{ cm}$$

Curved surface area = $2\pi rh = 2 \times \frac{22}{7} \times 7 \times 8 = 352 \text{ cm}^2$

3. Radius and slant height of a cone are 20 cm and 29 cm respectively. Find its volume.

Solution:

Given that r = 20 cm and ℓ = 29 cm

$$h = \sqrt{\ell^2 - r^2}$$

$$= \sqrt{29^2 - 20^2}$$

$$= \sqrt{841 - 400}$$

$= \sqrt{441}$

$h = 21$ cm

Volume of the cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 20 \times 20 \times 21$

$= 8800$ cm^3.

4. The perimeter of the ends of a frustum are 44 cm and 8.4π cm. If the depth is 14 cm., then find its volume.

Solution:

Given that $2\pi R = 44$ cm and $2\pi r = 8.4\,\pi$cm

$2 \times \frac{22}{7} \times R = 44$ $\qquad$ $2r = 8.4$

$R = \frac{44 \times 7}{2 \times 22}$ $\qquad$ $r = 4.2$ cm

$R = 7$ cm

Volume of the frustum $= \frac{1}{3}\pi h\,(R^2 + r^2 + Rr)$

$= \frac{1}{3} \times \frac{22}{7} \times 14\ (7^2 + 4.2^2 + 7 \times 4.2)$

$= \frac{44}{3}\ (49 + 29.4 + 17.64)$

$= \frac{44}{3} \times 96.04$

$= 1408.58$ cm^3

5. A solid wooden toy is in the form of a cone surmounted on a hemisphere. If the radii of the hemisphere and the base of the cone are 3.5 cm each and the total height of the toy is 17.5cm, then find the volume wooden used in the toy (Take $\pi = \frac{22}{7}$)

Solution:

Given that

<u>Hemisphere</u>

$r = 3.5$ cm

Volume $= \frac{2}{3}\pi r^3$

$= \frac{2}{3}\pi \times 3.5 \times 3.5 \times 3.5$

<u>Cone</u>

$h = 3.\ 5$cm

$h = 17.5 - 3.5$

$= 14$ cm

Volume $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi \times 3.5 \times 3.5 \times 14$

Volume of the wood = Volume of the hemisphere + Volume of the cone

$= \frac{2}{3}\pi \times 3.5 \times 3.5 \times 3.5 + \frac{1}{3}\pi \times 3.5 \times 3.5 \times 14$

$= \frac{1}{3}\ \pi \times 3.5 \times 3.5\ [2 \times 3.5 + 14]$

$$= \frac{1}{3} \times \frac{22}{7} \times 3.5 \times 3.5\ [7+14]$$

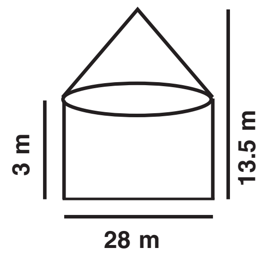
$$= \frac{1}{3} \times \frac{22}{7} \times 3.5 \times 3.5 \times 21$$

= 22 x 3.5 x 3.5

= 296.5 cu.cm

6. A tent is in the shape of a right circular cylinder surmounted by a cone. The total height and the diameter of the base are 13.5 m and 28 m. If the height of the cylindrical portion is 3 m, find the total surface area of the tent.

Solution:



Cone

h = 3m

2r = 28m

r = 14 m

CSA of the cylinder = $2\pi rh$

$= 2\pi \times 14 \times 3$

$= 84\pi$

Cylinder

h = 14 m

h = 13.5 - 3

= 10.5 m

$\ell = \sqrt{r^2 + h^2}$

$= \sqrt{14^2 + 10.5^2}$

$= \sqrt{196 + 110.25}$

$= \sqrt{306.25}$

= 17.5 m

CSA of the cone

$= \pi r\ell$

$= \pi \times 14 \times 17.5$ m

$= 245\pi$

Total surface area of the tent = CSA of the cylinder + CSA of the cone

$= 84\pi + 245\pi$

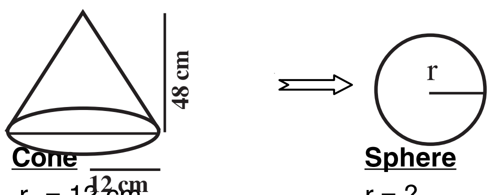
$= 329\pi$

$= 329 \times \frac{22}{7}$

= 1034 sq.m.

7. Using clay, a student made a right circular cone of height 48 cm and base radius 12 cm. Another student reshapes it in the form of a sphere. Find the radius of the sphere. **(Sep.12, March 14, June 14)** ★

Solution:



Cone

r_1 = 12 cm

h = 48 cm

Sphere

r = ?

Volume of the sphere = volume of the cone

ie $\frac{4}{3}\pi r^3 = \frac{1}{3}\pi r_1^2 h$

$4r^3 = r_1^2 h$

$4r^3 = 12 \times 12 \times 48$

$r^3 = \frac{12 \times 12 \times 48}{12}$

$r^3 = 12 \times 12 \times 12$

$r = 12$ cm

8. An iron right circular cone of diameter 8 cm and height 12 cm is melted and recast into spherical lead shots each of radius 4 mm. How many lead shots can be made?

Solution :

Let r and h be the radius and height of the cone. Let r_1 be the radius of the sphere.

Cone

$h = 12$ cm $= 120$ mm

$2r = 8$ cm

$r = 4$cm

$= 40$ mm

Sphere

$r_1 = 4$mm

$$\text{number of lead shots} = \frac{\text{Volume of the cone}}{\text{Volume of the sphere}}$$

$$= \frac{\frac{1}{3}\pi r^2 h}{\frac{4}{3}\pi r_1^3} = \frac{r^2 h}{4r_1^3}$$

$$n = \frac{40 \times 40 \times 120}{4 \times 4 \times 4 \times 4}$$

$= 750$

9. A cylindrical bucket of height 32 cm and radius 18 cm is filled with sand. The bucket is emptied on the ground and a conical heap of sand is formed. If the height of the conical heap is 24 cm, find the radius and slant height of the heap.

Solution:

Cylinder (bucket)

$h = 32$ cm

$r = 18$ cm

Volume of the cylinder $= \pi r^2 h$

$= \pi \times 18 \times 18 \times 32$

Cone (sand)

$h = 24$ cm

$r = ?$

Volume of the cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi \times r^2 \times 24$

Volume of the cone = Volume of the cylinder

$\frac{1}{3}\pi \times r^2 \times 24 = \pi \times 18 \times 18 \times 32$

$r^2 \times 8 = 18 \times 18 \times 32$

$r^2 = \frac{18 \times 18 \times 32}{8}$

$r^2 = 18 \times 18 \times 4$

$r = 18 \times 2$

$= 36$ cm

$\ell = \sqrt{h^2 + r^2}$

$= \sqrt{24^2 + 36^2}$

$= \sqrt{576+1296}$

$= \sqrt{1872}$

$= 12\sqrt{13}$ cm

10. A cylindrical shaped well of depth 20 m and diameter 14 m is dug. The dug out soil is evenly spread to form a cuboid-platform with base dimension 20 m x 14 m. Find the height of the platform.

Solution:

Cylinder (Well)	**Cuboid (Platform)**
h = 20 m	ℓ = 20 m
2r = 14 m	b = 14 m
r = 7 m	h_1 = ?
Volume = πr^2 h	Volume = ℓ bh
$= \frac{22}{7}$ x 7 x 7 x 20	= 20 x 14 x h
= 22 x 7 x 20	

Volume of the cuboid = Volume of the cylinder

20 x 14 x h = 22 x 7 x 20

$$h = \frac{22 \times 7 \times 20}{20 \times 14}$$

= 11 m

11. STATISTICS

1. Find co-efficient of variation 18, 20, 15, 12, 25

n = 5

$$\bar{x} = \frac{18+20+15+12+25}{5} = \frac{90}{5} = 18$$

x	$d = x - \bar{x}$	d^2
18	0	0
20	2	4
15	-3	9
12	-6	36
25	7	49
		98

$$\sigma = \sqrt{\frac{\sum d^2}{n}} = \sqrt{\frac{98}{5}} = \sqrt{19.6} \simeq 4.428$$

$$C.V. = \frac{\sigma}{\bar{x}} \times 100\% = \frac{4.428}{18} \times 100$$

C.V. = 24.6%

2. Find coefficient of variation of 20, 18, 32, 24, 26.

n = 5

$$\bar{x} = \frac{20+18+32+24+26}{5}$$

$$= \frac{120}{5} = 24$$

x	$d = x - \bar{x}$	d^2
20	-4	16
18	-6	36
32	8	64
24	0	0
26	2	4
		120

$$\sigma = \sqrt{\frac{\sum d^2}{n}} = \sqrt{\frac{120}{5}} = \sqrt{24} \simeq 4.9$$

$$\text{C.V} = \frac{\sigma}{\bar{x}} \times 100\% = \frac{4.9}{24} \times 100$$

C.V. = 20.4%

3. Find standard deviation of 20, 14, 16, 30, 21, 25.

$$\bar{x} = \frac{20+14+16+30+21+25}{6} = \frac{126}{6} = 21$$

x	$d = x - \bar{x}$	d^2
20	-1	1
14	-7	49
16	-5	25
30	9	81
21	0	0
25	4	16
		172

$$\sigma = \sqrt{\frac{\sum d^2}{n}} = \sqrt{\frac{172}{6}} = \sqrt{28.6}$$

$\sigma \simeq 5.3$

4. Find the standard deviation for 62, 58, 53, 50, 63, 52, 55

n = 7

$$\bar{x} = \frac{62+58+53+50+63+52+55}{7}$$

$$= \frac{393}{7} = 56$$

x	$d = x - \bar{x}$	d^2
62	6	36
58	2	4
53	-3	9
50	-6	36
63	7	49
52	-4	16
55	-1	1
		151

$$\sigma = \sqrt{\frac{\sum d^2}{n}} = \sqrt{\frac{151}{7}} = \sqrt{21.5}$$

$\sigma \simeq 4.9$

5. Find standard deviation for 10, 20, 15, 8, 3, 4

n = 6

$$\bar{x} = \frac{10+20+15+8+3+4}{6} = \frac{60}{6} = 10$$

x	d = x - 10	d^2
10	0	0
20	10	100
15	5	25
8	-2	4
3	-7	49
4	-6	36
		214

$$\sigma = \sqrt{\frac{\sum d^2}{n}} = \sqrt{\frac{214}{6}} = \sqrt{35.6}$$

$\sigma \simeq 5.9$

6. Find standard deviation of 38, 40, 34, 31, 28, 26, 34.

n = 7

$$\bar{x} = \frac{38+40+34+31+28+26+34}{7} = \frac{231}{7} = 33$$

x	d = x - $\bar{x}$	d^2
38	5	25
40	7	49
34	1	1
31	-2	4
28	-5	25
26	-7	49
34	1	1
		154

$$\sigma = \sqrt{\frac{\sum d^2}{n}} = \sqrt{\frac{154}{7}} = \sqrt{22}$$

$\sigma \simeq 4.69$

12. PROBABILITY

1. Three coins are tossed. Find the probability that either exactly two tails or atleast one head.

S = {(HHH, HHT, HTH, THH, TTT, TTH, THT, HTT}

n(S) = 8

Exactly two tails : A = {HTT, TTH, THT}, n (A) = 3

$$P(A) = \frac{3}{8}$$

Atleast one head : B = {HTT, THT, TTH, HHT, HTH, THH, HHH}

$$n(B) = 7, P(B) = \frac{7}{8}$$

$A \cap B$ = {HTT, TTH, THT}, $n(A \cap B) = 3$

$P(A \cap B) = \frac{3}{8}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B) \Rightarrow \frac{3}{8} + \frac{7}{8} - \frac{3}{8} = \frac{7}{8}$

2. A die is thrown twice. Find the probability that atleast one of two throws comes up with number 5.

S = {(1,1) ..... (6, 6)} n(S) = 36

5 in first throw : A = {(5, 1) (5, 2) (5, 3) (5,4) (5,5) (5,6)}

$n(A) = 6$, $P(A) = \frac{6}{36}$

5 in second throw : B = {(1, 5) (2, 5) (3, 5) (4, 5) (5, 5) (6, 5)}

$n(B) = 6$, $P(B) = \frac{6}{36}$

$A \cap B = \{(5,5)\} \Rightarrow n(A \cap B) = 1 \Rightarrow P(A \cap B) = \frac{1}{36}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B) \Rightarrow \frac{6}{36} + \frac{6}{36} - \frac{1}{36} = \frac{11}{36}$

3. Entertainment English vowels or letter T. Find the probability for the above.

S = {E, N, T, E, R, T, A I, N, M, E, N, T} N(S) = 13

English vowels : A = {E, E, A, I, E}, n (A) = 5

$P(A) = \frac{5}{13}$

Letter T : B = {T, T, T}, n(B) = 3

$P(B) = \frac{3}{13}$

$n(A \cap B) = 0$, $P(A \cap B) = 0$

$P(A \cup B) = P(A) + P(B) \Rightarrow \frac{5}{13} + \frac{3}{13} = \frac{8}{13}$

4. 52 cards in a packet,choose a card be spade card or king card.

n(S) = 52

Spade card : A n(A) = 13 , $P(A) = \frac{13}{52}$

King card : B n(B) = 4, $P(B) = \frac{4}{52}$

$n(A \cap B) = 1$, $P(A \cap B) = \frac{1}{52}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$\frac{13}{52} + \frac{4}{52} - \frac{1}{52} = \frac{16}{52} = \frac{4}{13}$$

5. 10 white, 6 red, 10 black, balls are in a bag. Choose a ball to be white or red colour ball.

n(S) = 10 + 6 + 10 = 26

White ball : A n(A) = 10, $P(A) = \frac{10}{26}$

Red ball : B n(B) = 6 , $P(B) = \frac{6}{26}$

$$P(A \cup B) = P(A) + P(B) = \frac{10}{26} + \frac{6}{26} = \frac{16}{26} = \frac{8}{13}$$

6. If a die is rolled twice find the probability of getting an even number in the first time or a total of 8.

S = {(1, 1) ...... (1,6) } n (S) = 36

Even numbers: A = { (2, 1) (2, 2) (2, 3) (2,4) (2,5) (2,6)
(4, 1) (4, 2) (4, 3) (4,4) (4,5)(4,6)
(6, 1) (6, 2) (6, 3) (6,4) (6,5) (6,6)}

$n(S) = 18,\ P(A) = \frac{18}{36}$

Total of 8 : B = {(2, 6) (3,5) (4,4) (5,3) (6,2)}

$n(B) = 5, \quad P(B) = \frac{5}{36}$

$A \cap B = \{(2, 6)\ (4, 4)\ (6, 2)\}\ n(A \cap B) = 3, \quad P(A \cap B) = \frac{3}{36}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$= \frac{18}{36} + \frac{5}{36} - \frac{3}{36} = \frac{20}{36} = \frac{5}{9}$$

7. Probability a new car for its design is 0.25, the probability of getting a award for use of fuel is 0.35. Find the probability i) Atleast one ii) get only one of the awards.

$P(A) = 0.25,\ P(B) = 0.35, \quad P(A \cap B) = 0.15$

i) $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.25 + 0.35 - 0.15$
$= 0.45$

ii) $P(\overline{A} \cap B) + P(A \cap \overline{B}) = [P(A) - P(A \cap B)] + [P(B) - P(A \cap B)]$

= [0.25 – 0.15] + [0.35 – 0.15[

= 0.10 + 0.20

= 0.3

8. Probability for admission in Medical college is 0.16, Admission in Enginnering college is 0.24, both is 0.11. Find the probability of i) atleast one ii) Medical or Engineering college.

$P(A) = 0.16,\ P(B) = 0.24, \quad P(A \cap B) = 0.11$

i) $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.16 + 0.24 - 0.11$
$= 0.40 - 0.11 = 0.29$

ii) $P(A \cap \overline{B}) + P(\overline{A} \cap B) = P(A) - P(A \cap B) + P(B) - P(A \cap B)$

= 0.16 – 0.11 + 0.24 – 0.11

= 0.05 + 0.13 = 0.18

9. A card from deck of 52 cards. Find the probability of getting a King or a heart or a red card.

n(S) = 52

King	Heart	Red card
n(A) = 4,	n(B) = 13,	n(C) = 26,
$P(A) = \frac{4}{52}$	$P(B) = \frac{13}{52}$	$P(C) = \frac{26}{52}$

$n(A \cap B) = 1 \quad n(B \cap C) = 13 \quad n(A \cap C) = 2 \quad n(A \cap B \cap C) = 1$

$P(A \cap B) = \frac{1}{52}$ $P(B \cap C) \frac{13}{52}$ $P(A \cap C) = \frac{2}{52}$ $P(A \cap B \cap C) = \frac{1}{52}$

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$$

$$= \frac{4}{52} + \frac{13}{52} + \frac{26}{52} - \frac{1}{52} - \frac{13}{52} - \frac{2}{52} + \frac{1}{52} = \frac{28}{52} = \frac{7}{13}$$

10. A bag contains 10 white, 5 black, 3 green and 2 red balls. Find the probability is a white or black or green

$n(S) = 10 + 5 + 3 + 2 = 20$

White : A — Black : B — Green : C

$n(A) = 10$ — $n(B) = 5$ — $n(C) = 3$

$P(A) = \frac{10}{20}$ — $P(B) = \frac{5}{20}$ — $P(C) = \frac{3}{20}$

$$P(A \cup B \cup C) = P(A) + P(B) + P(C)$$

$$= \frac{10}{20} + \frac{5}{20} + \frac{3}{20} = \frac{18}{20}$$

$$= \frac{9}{10}$$

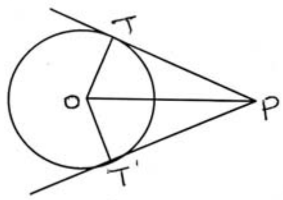
11. $P(A) = \frac{4}{5}$, $P(B) = \frac{2}{3}$, $P(C)\ \frac{3}{7}$, $P(A \cap B) = \frac{8}{15}$, $P(B \cap C) = \frac{2}{7}$, $P(A \cap C) = \frac{12}{35}$, $P(A \cap B \cap C) = \frac{8}{35}$,

$P(A \cup B \cup C) = ?$

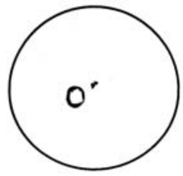
$$P(A \cup B \cup C) = \frac{4}{5} + \frac{2}{3} + \frac{3}{7} - \frac{8}{15} - \frac{2}{7} - \frac{12}{35} + \frac{8}{35}$$

$$= \frac{84 + 70 + 45 - 56 - 30 - 36 + 24}{105} = \frac{101}{105}$$

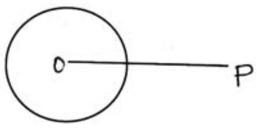
9. GEOMETRY



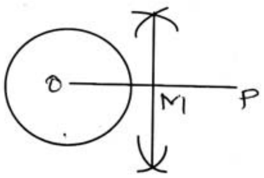
Step 1: Draw the rough diagram - write only the two measurements (raduis and OP length) in the rough diagram



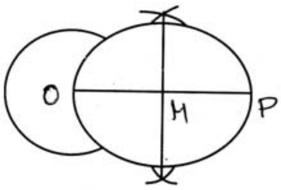
Step 2 : With O as the centre draw the first circle of radius given.



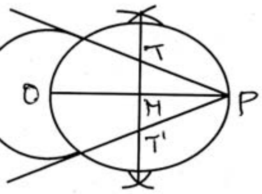
Step 3: Mark a point P at a distance of OP given.



Step 4: Draw the perpendicular bisector of OP.



Step 5: With M as centre and MO as radius draw the second circle.



Step 6 : T and T' be the point of intersection of the two circles. Join PT and PT'. Measure the lengths of PT and PT' and write. (Eg. 6.3 cm)

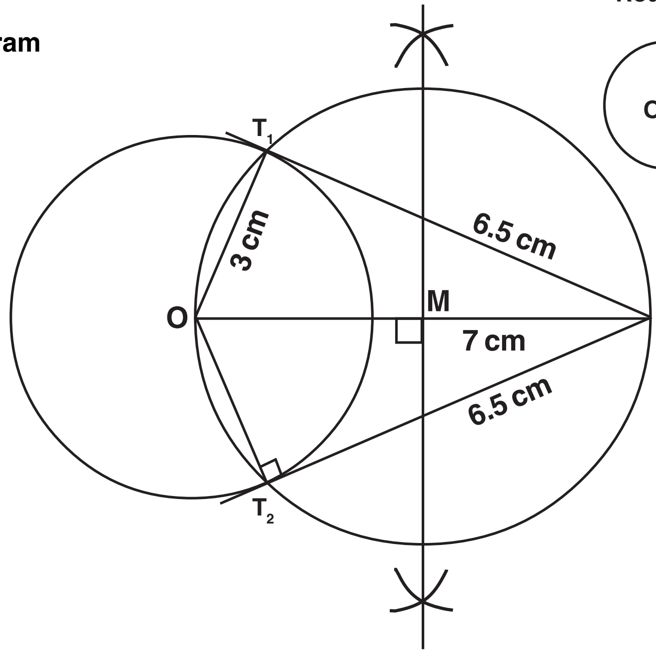
Note: i) Do not write the verifiction if not asked for

ii) Two tangents can be drawn to a circle from an external point.

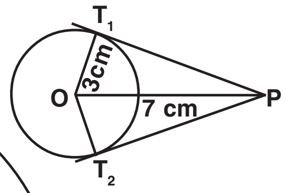
iii) Diameters subtend 90 degree on the circumferene of a circle.

1. Draw a circle of radius 3cm from an external point 7 cm away from its centre, construct the pair of tangents to the circle and measure their lengths.

Fair diagram

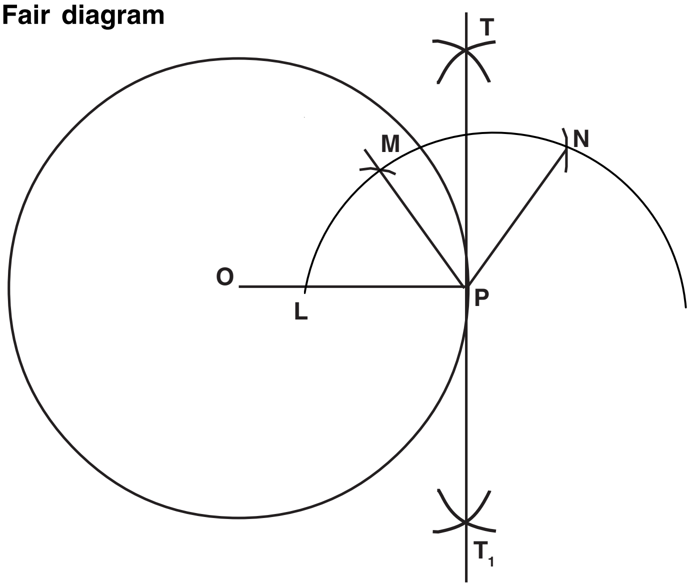


Rough Diagram

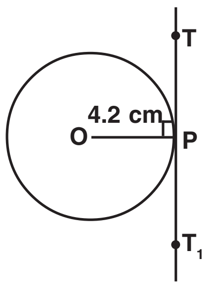


2. Draw a circle of radius 4.2cm, and take any point on the circle. Draw the tangent at that point using the centre.

Fair diagram



Rough Diagram



Construction

1) With O as the centre dra a circle of radius 4.2 cm.

2) Take a point P on the circle and join OP.

3) Draw an arc of a circle with centre at P cutting OP at L.

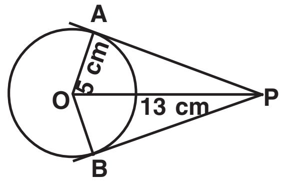
4) Mark M and N on the arc such that $\overline{LM} = \overline{MN}$ = LP

5) Draw the bisector PT of the angle $\angle$MPN

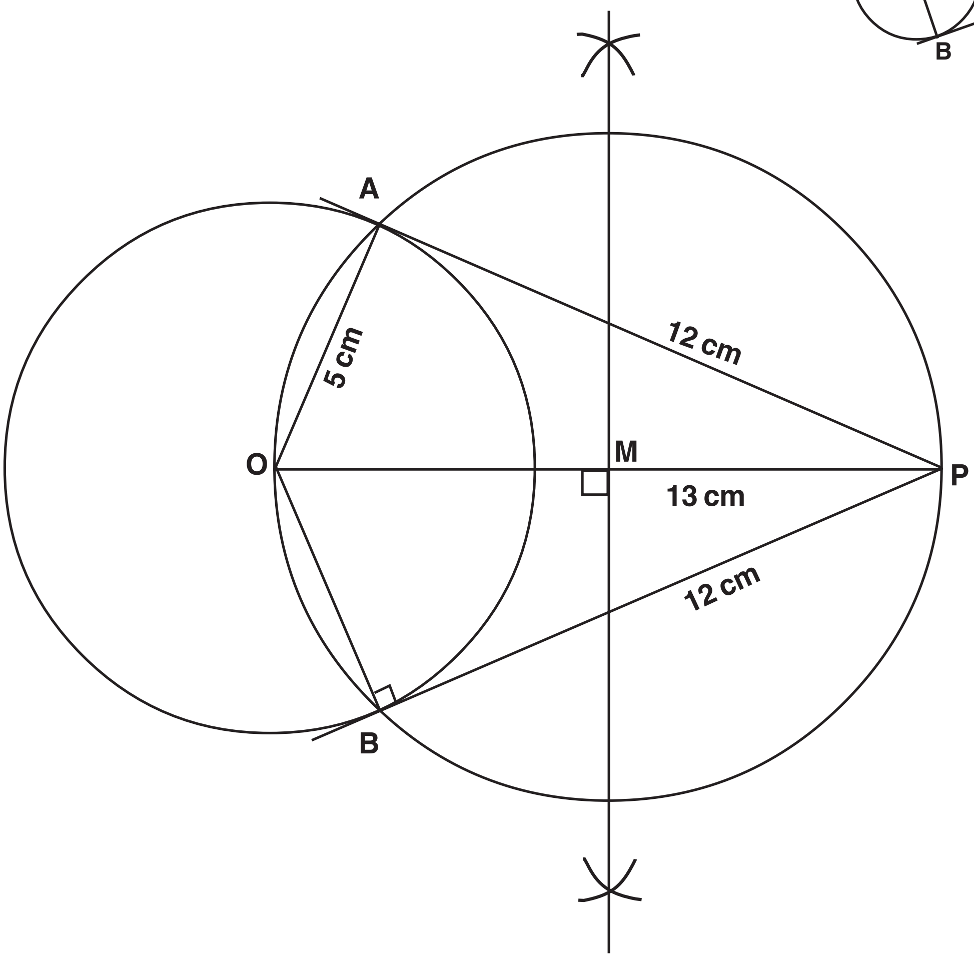
6) Extend TP to T to get the tangent.

3. Draw a circle of diameter 10cm. From a point 13 cm away from its centre, draw the two tangents PA and PB to the circle, and measure their lengths.

Rough Diagram



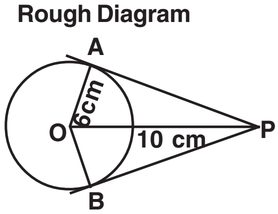
Fair Diagram

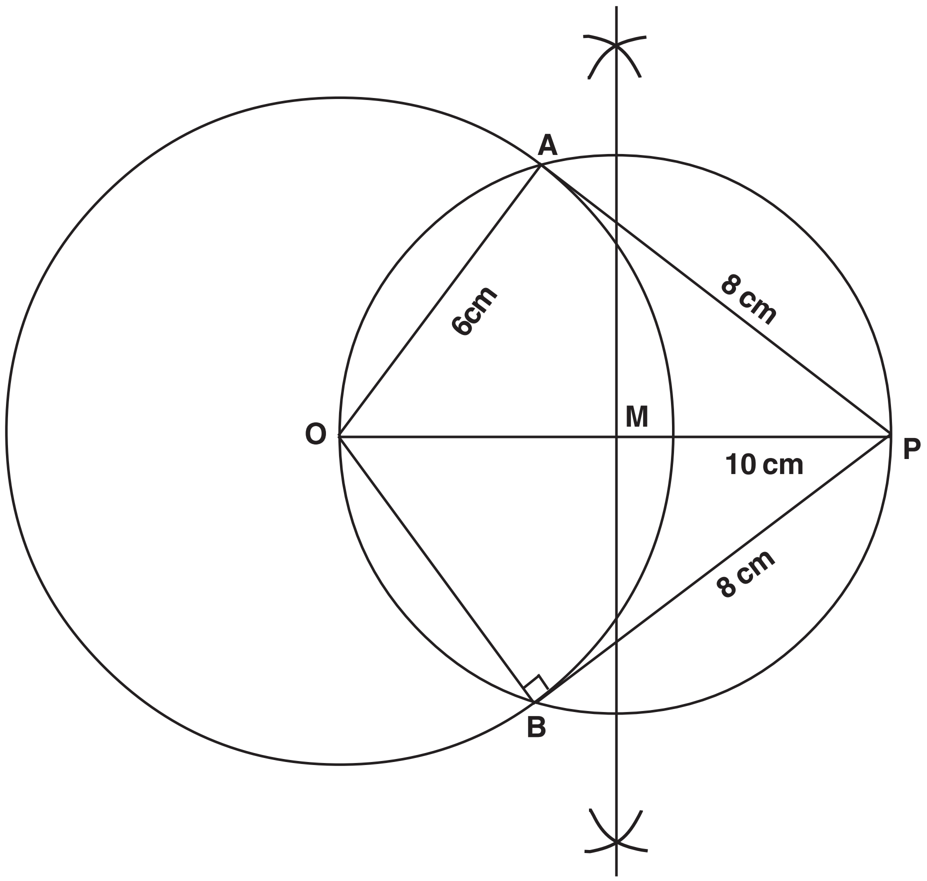


4. Draw the two tangents from a point which is 10 cm away from the centre of a circle of radius 6 cm. Also measure the lengths of the tangents.

Fair Diagram

Rough Diagram

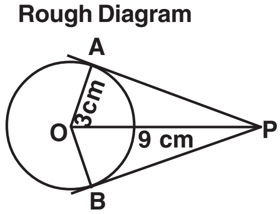


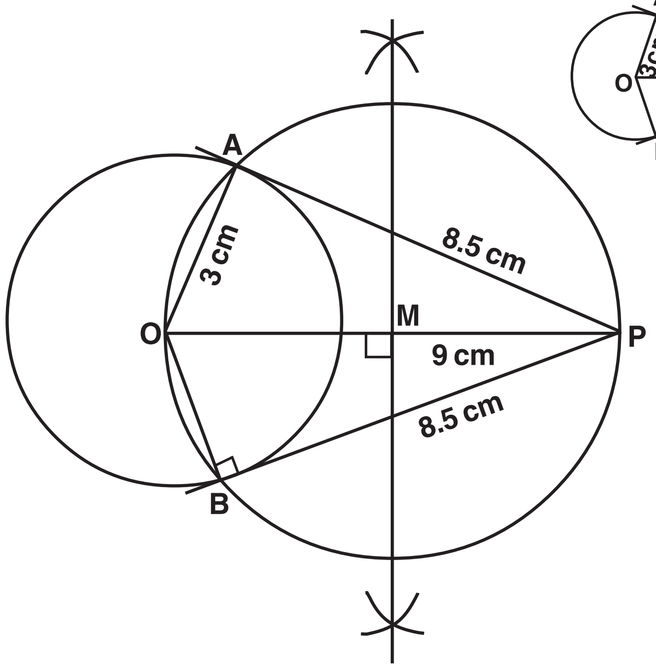


5. Take a point which is 9 cm away from the centre of the circle of radius 3 cm and draw the two tangents to the circle from that point.

Fair Diagram

Rough Diagram



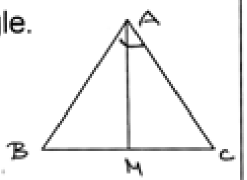


CONSTRUCTION OF TRIANGLES

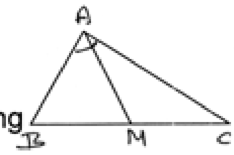
If
Base
Vertical Angle and
Altitude are given.

If
Base
Vertical Angle and
Median are given.

Angle BAC-Vertical Angle.
BC-Base Line.
AM-Altitude.
The perpendicular line drawn through vertex A to the base line is altitude.



BAC-Vertical Angle.
BC-Base Line.
AM-Median.
The line drawn joining vertex A and the midpoint of the base line BC is called Median.



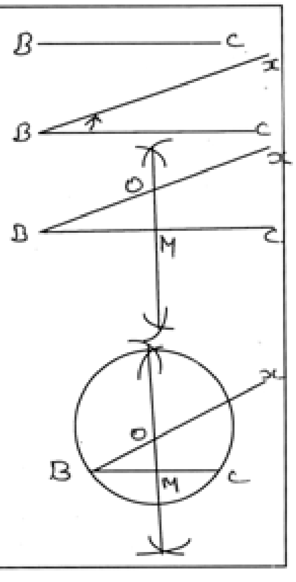
Step: 1.Draw the base line BC.
Step: 2. Subtract the given vertical angle from 90 (Eg.90-60=30)

Step: 3. Through B draw BX such that angle CBX=30.

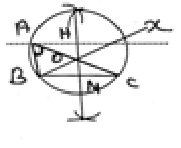
Step: 4. Draw theperpndicular bisector of BC intersecting BX at O and BC at M.

Step: 5. With O as centre and OB as radius draw the circle.

Note: Step 1 to 5 are common for altitude and median given.



Step: 6. On the perpendicular bisector MO, mark a point H such that the lengthof the altitude given.

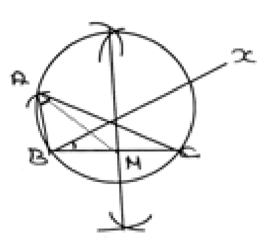


Step: 7. Draw the line AHA' parallel to BC meeting the circle at A and A'.

Step: 8. Join AB and AC and get the required triangle.

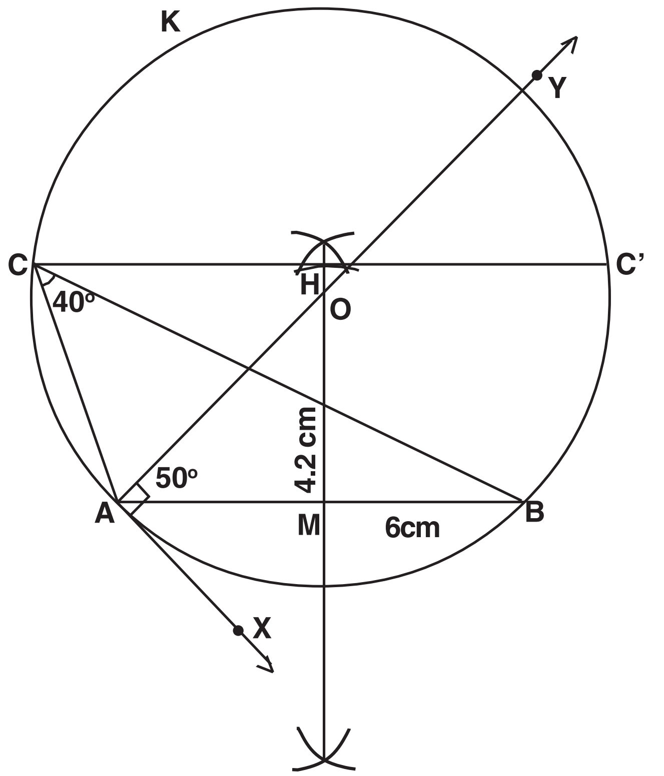
Step: 6. With M as centre and the length of given median as radius draw an arc in such a way cutting the circle at A and A'

Step: 7. Join AB , AC and AM and get the required triangle.

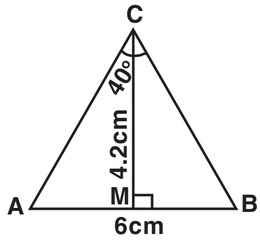


1. Construct a $\triangle ABC$ such that AB = 6cm $\angle C = 40^\circ$ and the altitude from C to AB is of length 4.2 cm.

Fair Diagram

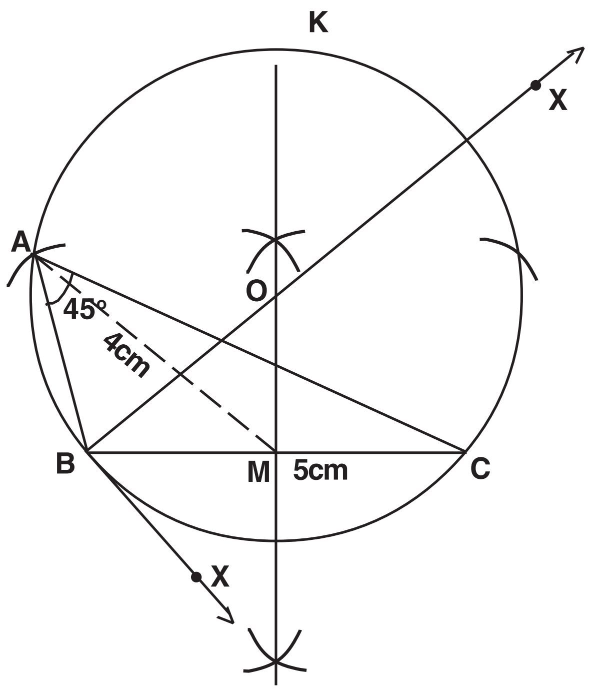


Rough Diagram

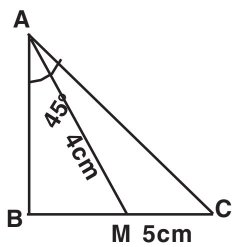


2. Construct a $\triangle ABC$ such that BC = 5cm $\angle A = 45^\circ$ and the medium from A to BC is 4 cm.

Fair Diagram

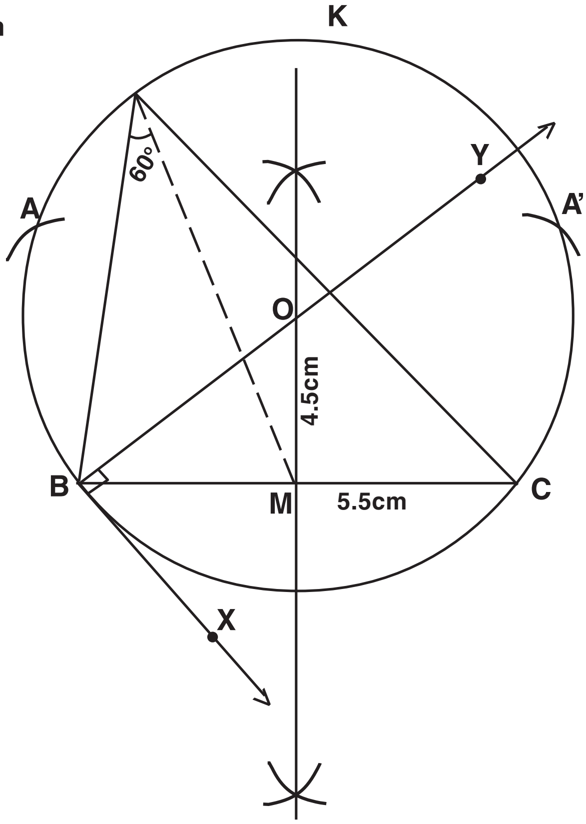


Rough Diagram

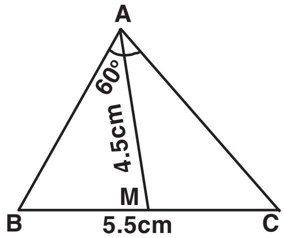


3. Construct a $\triangle ABC$ in which BC = 5.5 cm $\angle A = 60^{o}$ and the median AM from the vertex A is 4.5 cm.

Fair diagram

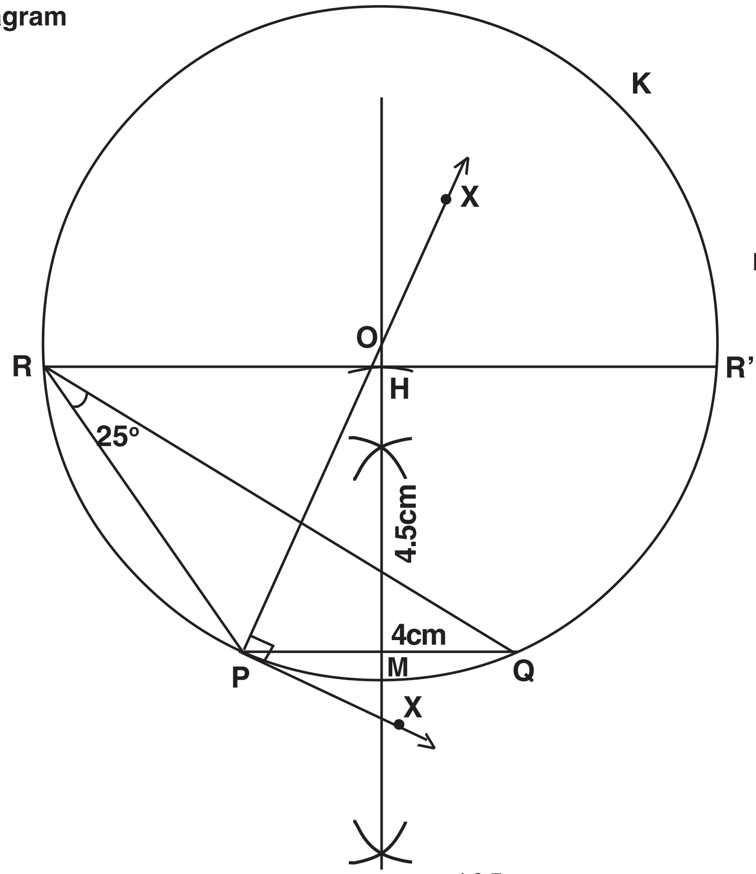


Rough Diagram

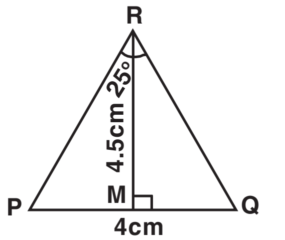


4. Construct a $\triangle PQR$ such that PQ = 4 cm $\angle R = 25^{o}$ and the altitude from R to PQ is 4.5 cm.

Fair Diagram

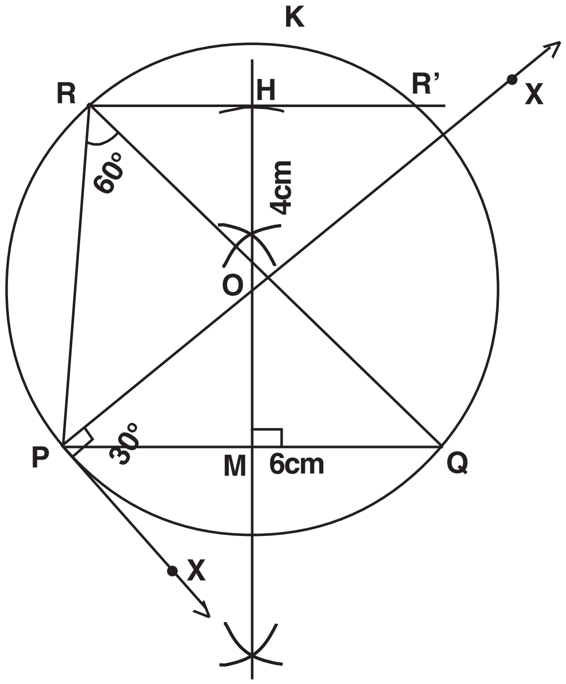


Rough Diagram

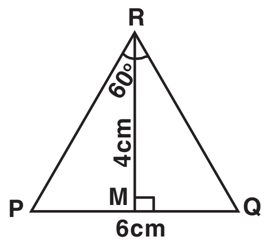


5. Construct a ΔPQR in which the base PQ = 6cm $\angle R = 60^{\circ}$ and the altitude from R to PQ is 4 cm.

Fair Diagram

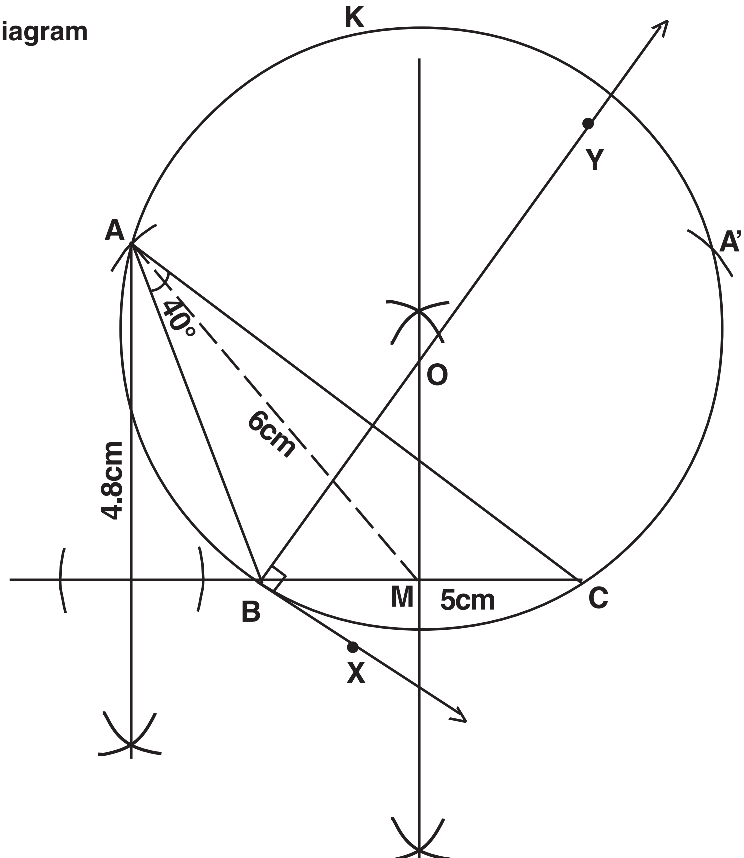


Rough Diagram

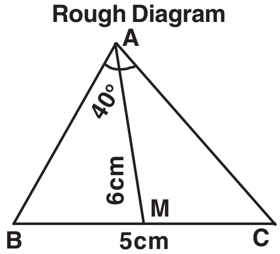


6. Construct a ΔABC in which the base BC = 5cm $\angle BAC = 40^{\circ}$ and the median from A to BC is 6 cm. Also measure the length of the altitude from A.

Fair Diagram



Rough Diagram



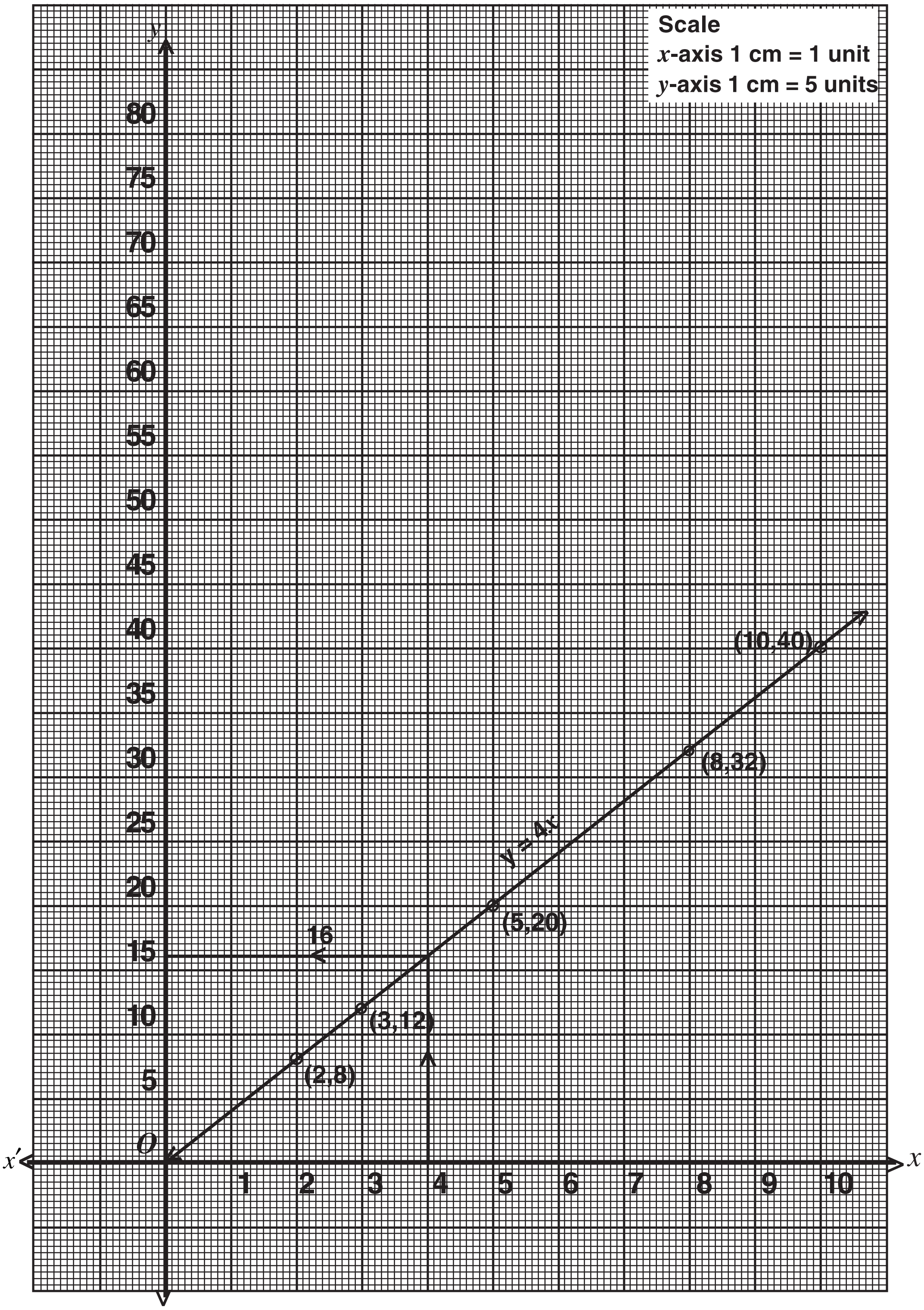
10. GRAPHS

1. Draw a graph for the following table and identify the variation

x	2	3	5	8	10
y	8	12	20	32	40

Hence, find the value of y when x=4.

Solution : When x = 4 , y = 16



2. A cyclist travels from a place A to a place B along the same route at a uniform speed on different days. The following table gives the speed of his travel and the corresponding time he took to cover the distance.

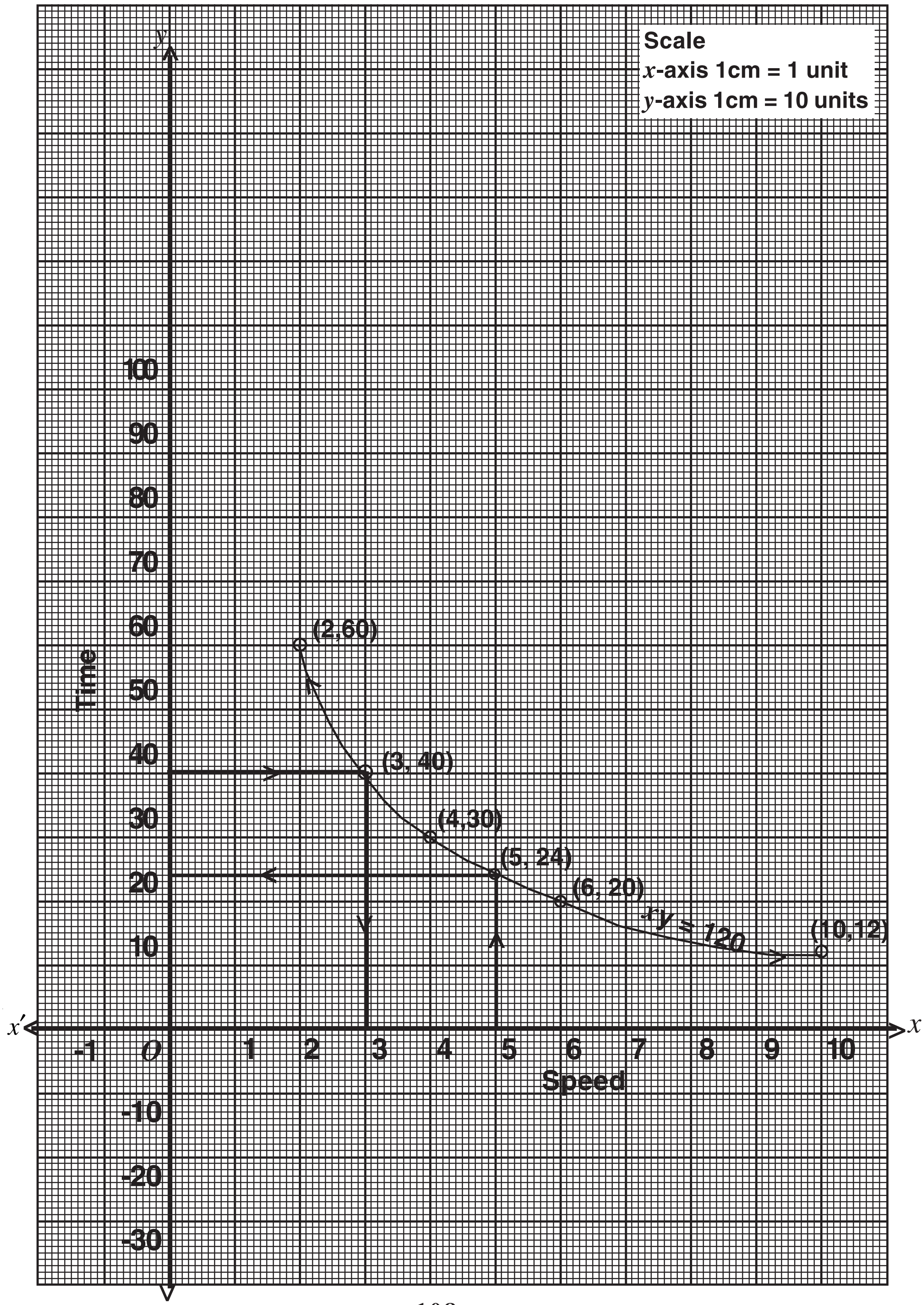
Speed in km/hr (x)	2	4	6	10	12
Time in hrs (y)	60	30	20	12	10

Draw the speed-time graph and use it to find

i) the number of hours he will take if he travels at a speed of 5km/hr

ii) the speed with which he should travel if he has to cover the distance in 40 hrs.

Solution: i) When x = 5, y = 24,   ii) When y = 40, x = 3



3. The following table gives the cost and number of notebooks bought.

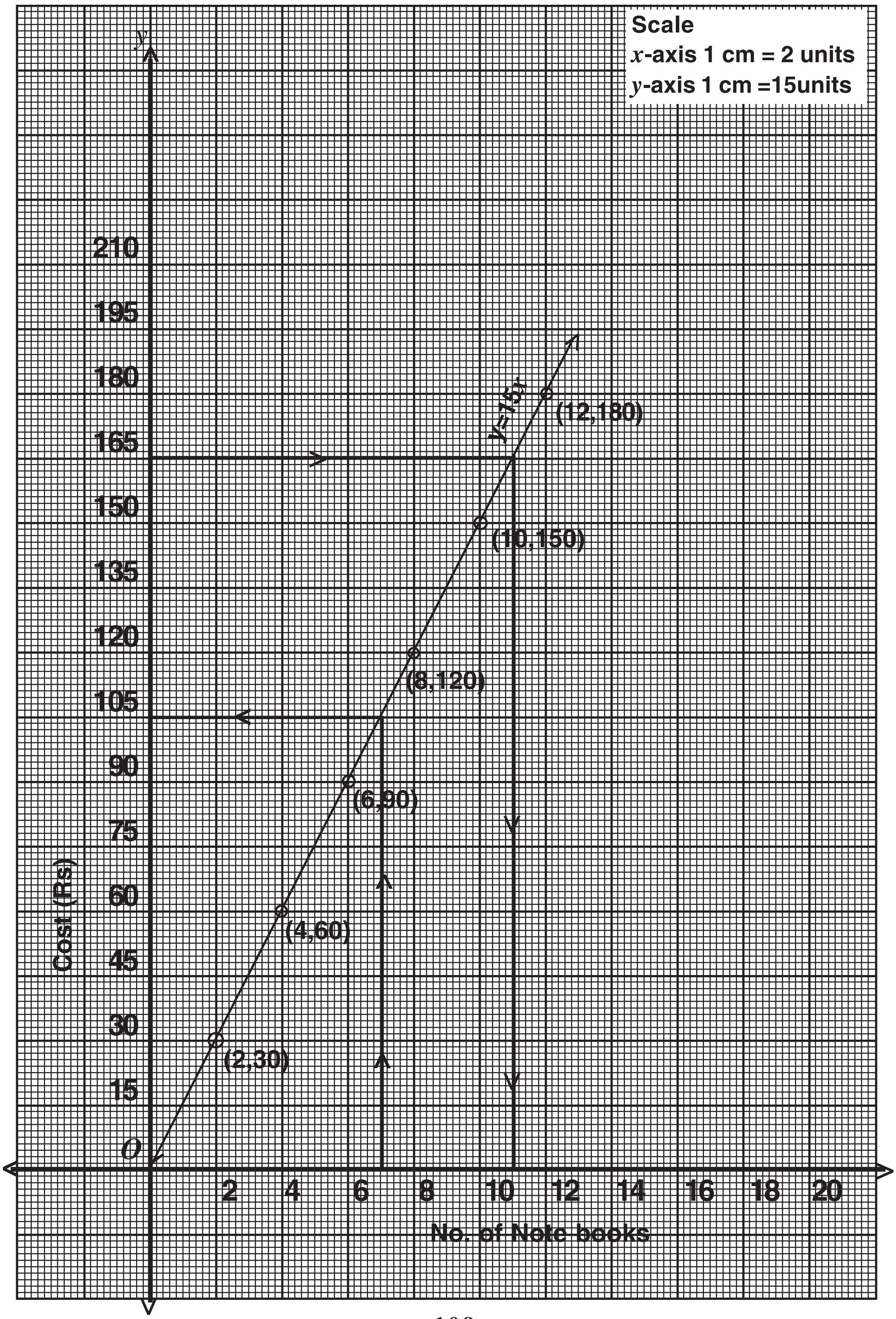
No.of note books (x)	2	4	6	8	10	12
Cost Rs. (y)	30	60	90	120	150	180

Draw the graph and hence i) Find the cost of seven note books.

ii) How many note books can be bought for Rs. 165.

Solution: i) The cost of seven note books = Rs. 105

ii) Number of note books that can be bought for Rs. 165 = 11



4.

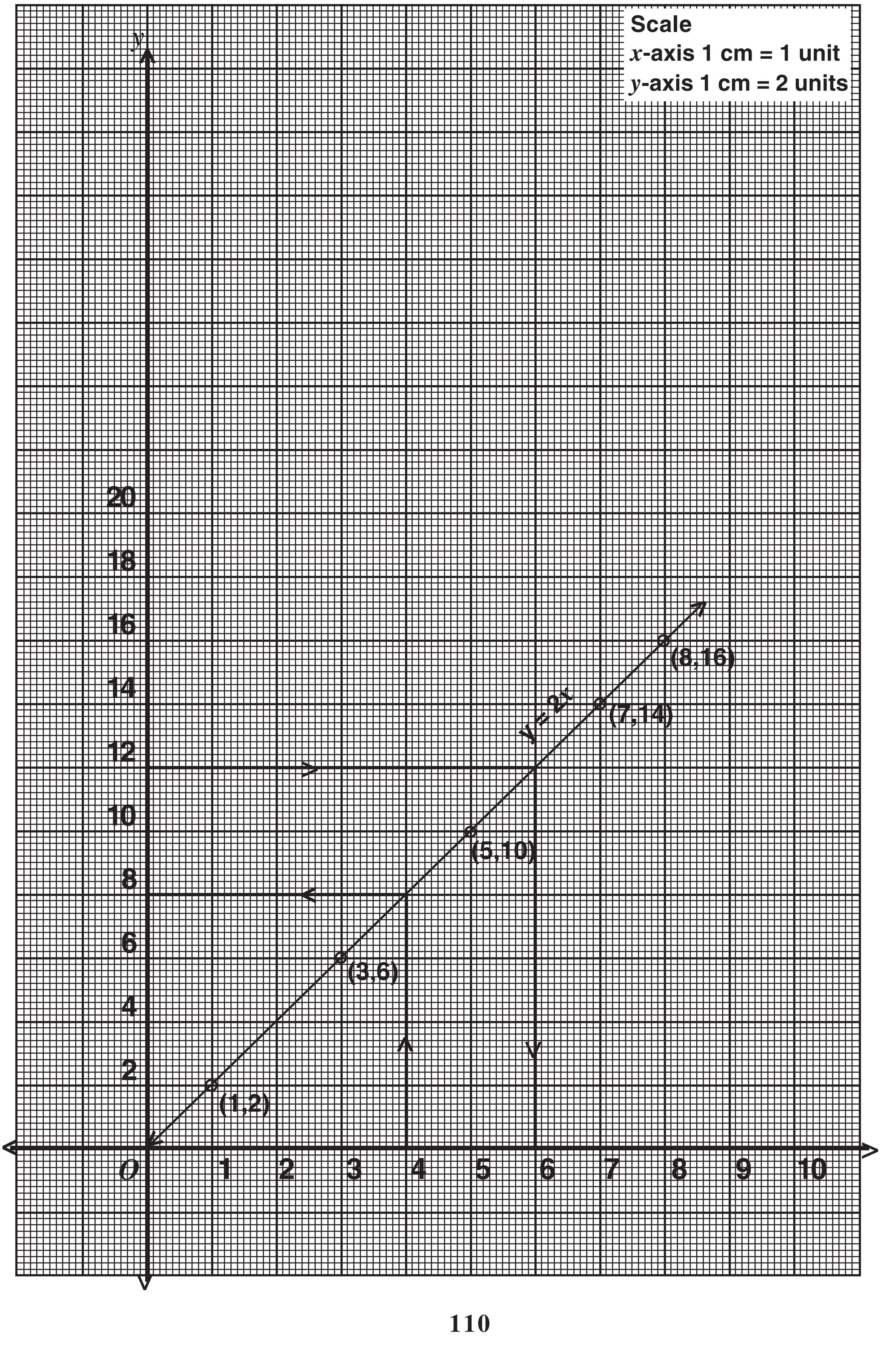
x	1	3	5	7	8
y	2	6	10	14	16

Draw the graph for the above table and hence find

i) the value of y if $x = 4$ ii) the value of x if y = 12

Solution : i) When x = 4, y = 8

ii) When y = 12, x = 6

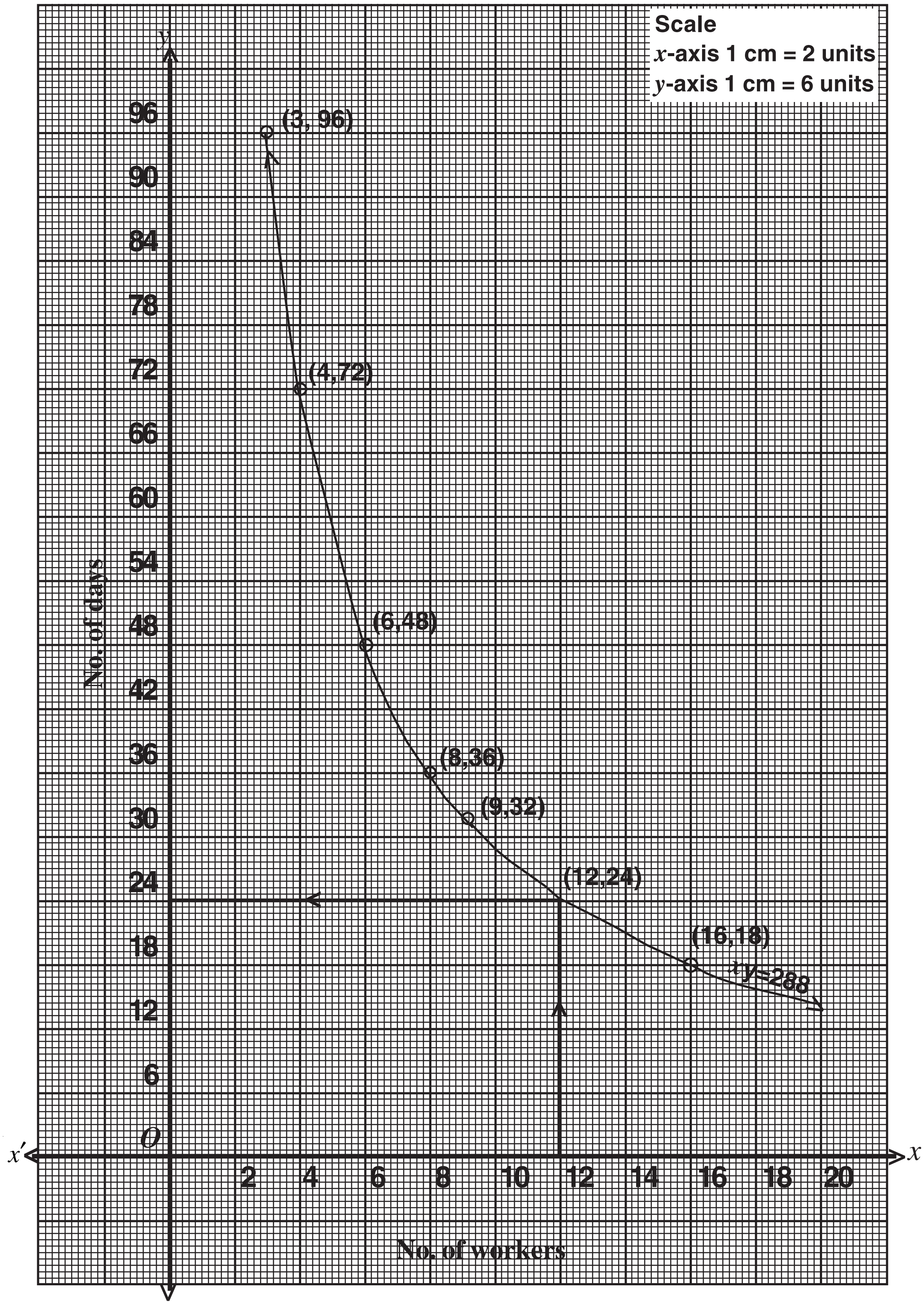


5.

No.of workers (x)	3	4	6	8	9	16
No. of days (y)	96	72	48	36	32	18

Draw the graph for the data given in the table. Hence find the number of days taken by 12 workers to complete the work.

Solution : Number of days to complete the work by 12 workers = 24

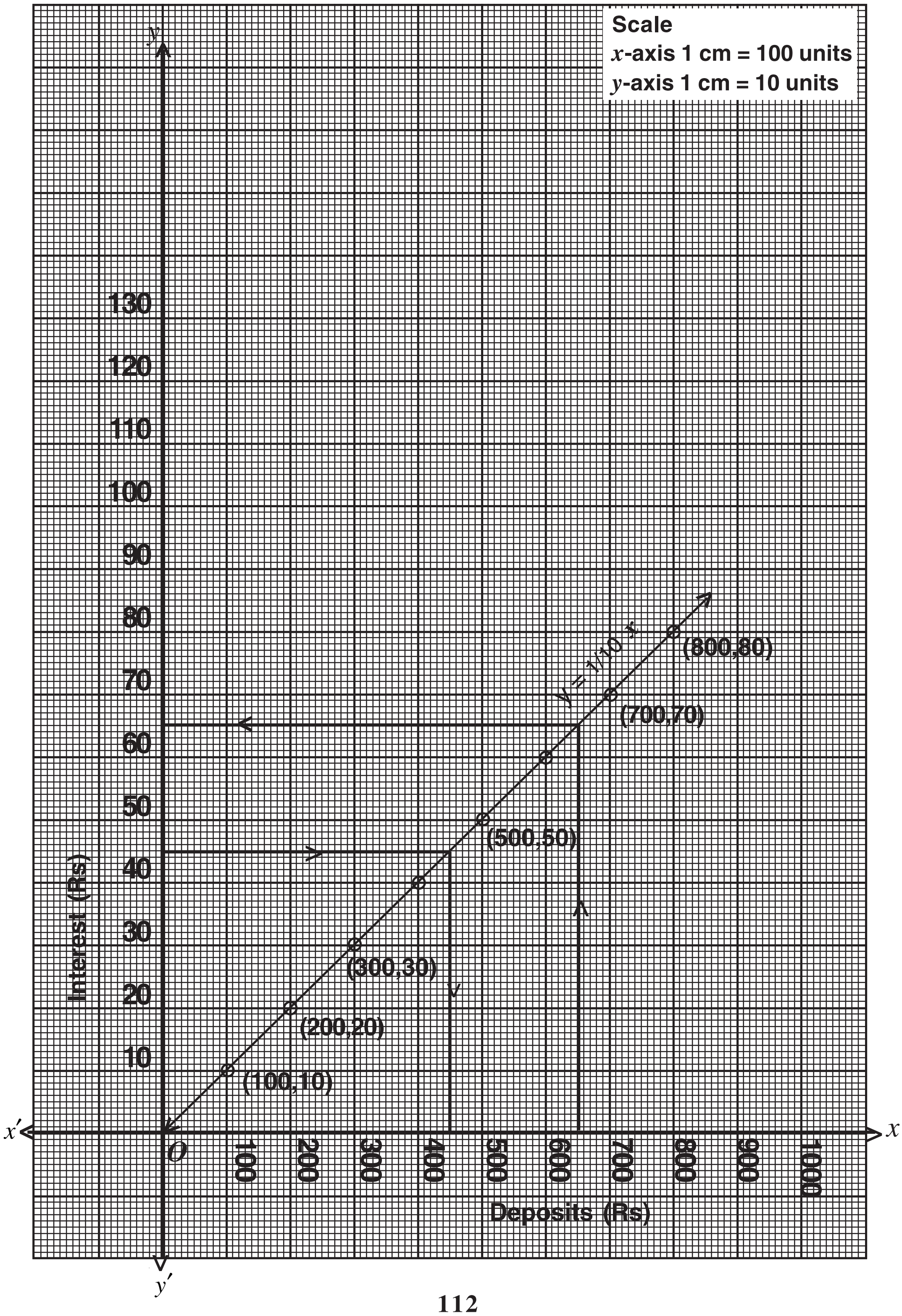


1. A bank gives 10% S.I. on deposits for senior citizens. Draw the graph for the relation between the sum deposited and the interest earned for one year. Hence find i) the interest on the deposit of Rs. 650 ii) the amount to be deposited to earn an interest of Rs. 45.

Deposit (x)	100	200	300	400	500	600	700	800
Interest (y)	10	20	30	40	50	60	70	80

Solution: i) The interest for the deposit of Rs.650 is Rs. 65

ii) The amount to be deposited to earn an interest of Rs.45 is Rs. 450



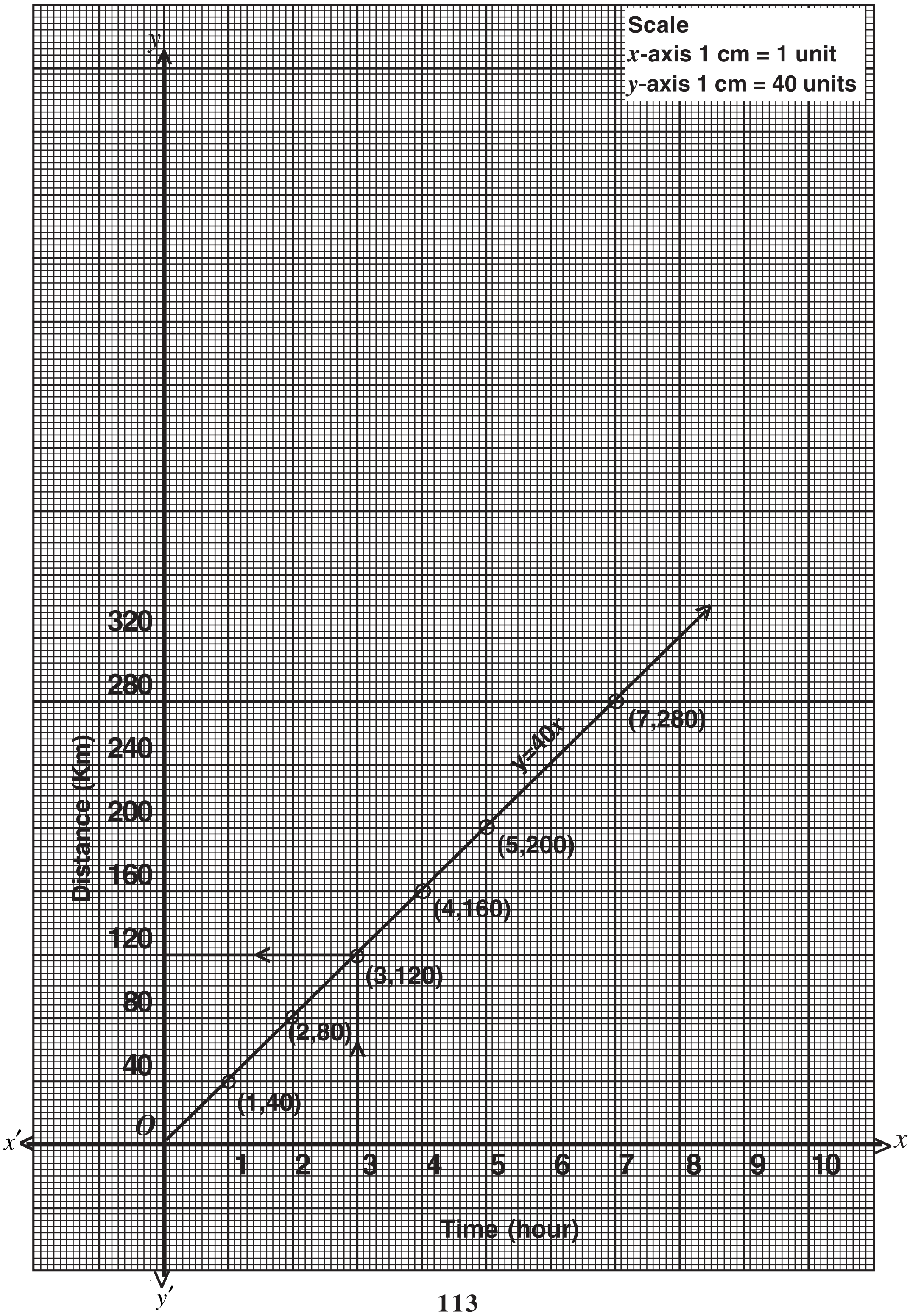
2. A bus travels at a speed of 40km/hr. Write the distance-time formula and draw the graph of it. Hence, find the distance travelled in 3 hours. **(June 13, June 14)**

Solution:

We can form the following table from the given information.

Time (hr)	1	2	3	4	5	6	7
Distance(km)	40	80	120	160	200	240	280

Solution : From the graph, the distance braveled in 3 hours in 120 km.



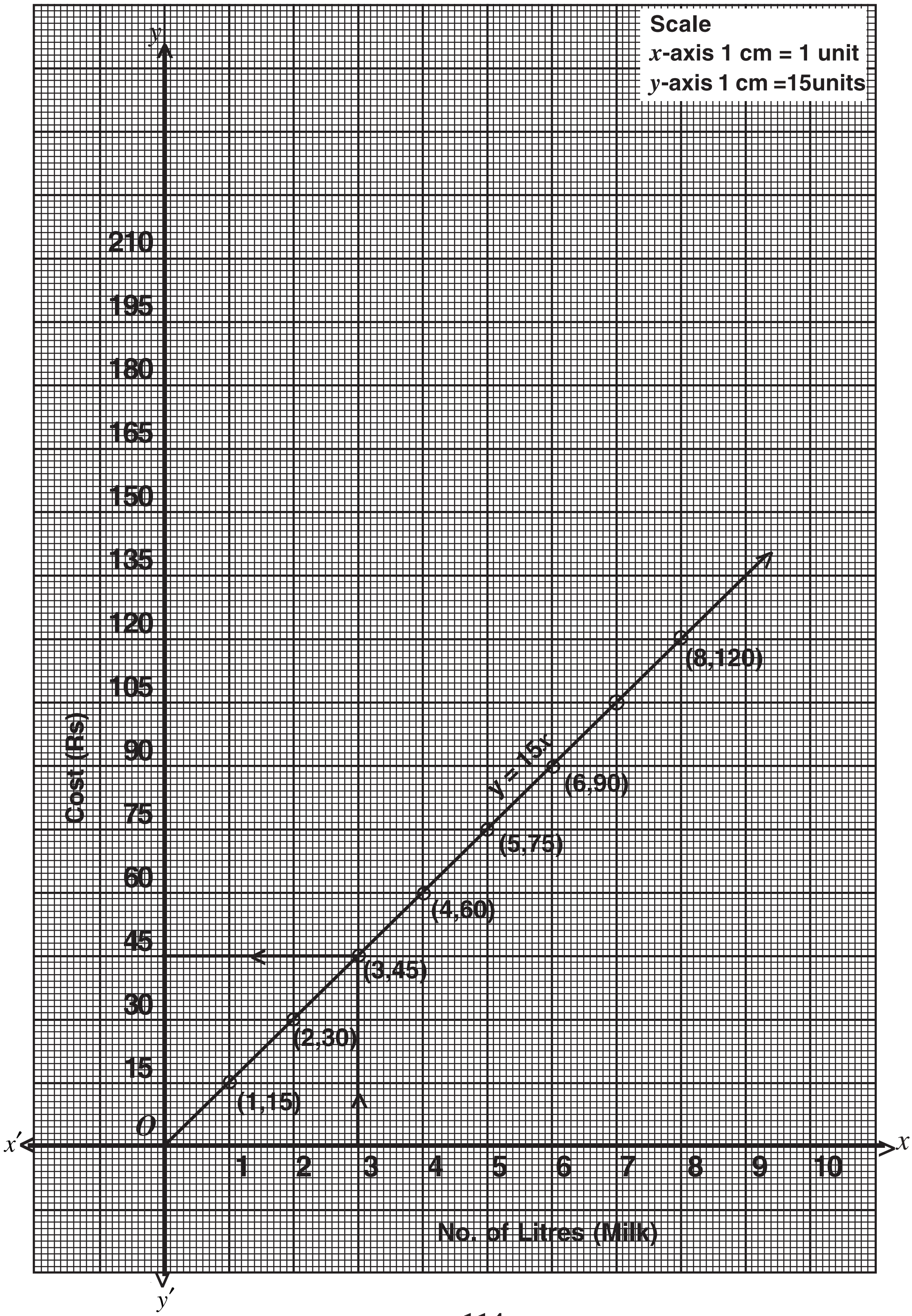
3. The cost of the milk per litre is Rs. 15. Draw the graph for the relation between the quantity and cost. Hence find i) the proportionality constant. ii) the cost of 3 litres of milk

Solution:

We can form the following table from the given information.

No.of litres	1	2	3	4	5	6	7
Cost	15	30	45	60	75	90	105

Solution: i) The proportionality constant = 15 ii) The cost of 3 litres of milk = Rs. 45



4. Draw the graph of $xy = 20$, x, $y > 0$. Use the graph to find y when $x = 5$, and to find x when $y = 10$.

Solution:

We can form the following table from the given information.

x	1	2	4	5	10	20
y	20	10	5	4	2	1

Solution : When x = 5 then y = 4

When y = 10 then x = 2