TWO MARKS QUESTIONS

1. SETS AND FUNCTIONS

1. If A = {4, 6, 7, 8, 9}, B = {2, 4, 6}, C = {1, 2, 3, 4, 5, 6} then find $A\cup(B\cap C)$.

Solution :

$A = \{4, 6, 7, 8, 9\}, B = \{2, 4, 6\}, C = \{1, 2, 3, 4, 5, 6\}$

$B\cap C = \{2, 4, 6\}\cap \{1, 2, 3, 4, 5, 6\}$

$= \{2, 4, 6\}$

$\therefore\ A\cup(B\cap C) = \{4, 6, 7, 8, 9\}\cup\{2, 4, 6\}$

$= \{2, 4, 6, 7, 8, 9\}$

2. A = {10, 15, 20, 25, 30, 35, 40, 45, 50}, B = {1, 5, 10, 15, 20, 30}, C = {7,8, 15, 20, 35, 45, 48}
Find $A\backslash(B\cap C)$.

Solution:

$(B\cap C) = \{1, 5, 10, 15, 20, 30\}\cap\{7,8, 15, 20, 35, 45, 48\}$

$= \{15, 20\}$

$A\backslash(B\cap C) = \{10, 15, 20, 25, 30, 35, 40, 45, 50\} \backslash \{15, 20\}$

$= \{10, 25, 30, 35, 40, 45, 50\}$

3. Let P = {a, b, c}, Q = {g, h, x, y} and R = {a, e, f, s} find $R\backslash(P\cap Q)$

Solution:

$P\cap Q = \{a, b, c\} \cap \{g, h, x, y\} = \{\ \}$

$R\backslash(P\cap Q) = \{a, e, f, s\} \backslash \{\ \} = \{a, e, f, s\}$

4. If $\cup$ = {4, 8, 12, 16, 20, 24, 28}, A = {8, 16, 24} and B = {4, 16, 20, 28} then find $(A\cup B)'$ and $(A\cap B)'$.

Solution :

$A\cup B = \{8, 16, 24\} \cup \{4, 16, 20, 28\}$

$= \{4, 8, 16, 20, 24, 28\}$

$(A\cup B)' = \cup \backslash (A\cup B)$

$= \{ 4, 8, 12, 16, 20, 24, 28\} \backslash \{4, 8, 16, 20, 24, 28\}$

$= \{12\}$

$A\cap B = \{8, 16, 24\} \cap \{4, 16, 20, 28\}$

$= \{16\}$

$\therefore (A\cap B)' = \cup \backslash (A\cap B)$

$= \{4, 8, 12, 16, 20, 24, 28\} \backslash \{16\}$

$= \{4, 8, 12, 20, 24, 28\}$

5. A = {-10, 0, 1, 9, 2, 4, 5}, B = {-1, -2, 5, 6, 2, 3, 4} Verify that set intersection is commutative. Also verify it.

Solution:

$A\cap B = B\cap A$

$A\cap B = \{-10, 0, 1, 9, 2, 4, 5\} \cap \{-1, -2, 5, 6, 2, 3, 4\}$

$= \{2, 4, 5\}$ ---- (1)

$B\cap A = \{-1, -2, 5, 6, 2, 3, 4\} \cap \{-10, 0, 1, 9, 2, 4, 5\}$

$= \{2, 4, 5\}$ ---- (2)

(1) = (2)

$A\cap B = B\cap A$

6. If A = {4, 6, 7, 8, 9}, B = {2, 4, 6} and C = {1, 2, 3, 4, 5, 6} then find $A \cap (B \cup C)$.

Solution:

$B \cup C$ = {2, 4, 6} $\cup$ {1, 2, 3, 4, 5, 6}
= {1, 2, 3, 4, 5, 6}

$A \cap (B \cup C)$ = {4, 6, 7, 8, 9} $\cap$ {1, 2, 3, 4, 5, 6}
= {4, 6}

7. Verify the commutative property of set intersection for A = {ℓ, m, n, o, 2, 3, 4, 7} and B = {2, 5, 3, -2, m,n, o, p}.

$A \cap B = B \cap A$

Solution:

$A \cap B$ = {ℓ, m, n, o, 2, 3, 4, 7} $\cap$ {2, 5, 3, -2, m,n, o, p}.
= {m,n, o, 2, 3} ----- (1)

$B \cap A$ = {2, 5, 3, -2, m,n, o, p} $\cap$ {ℓ, m, n, o, 2, 3, 4, 7}
= {m, n, o, 2, 3} ----- (2)

(1) = (2)

$A \cap B = B \cap A$.

8. For A = {5, 10, 15, 20}, B = {6, 10, 12, 18, 24} and C = {7, 10, 12, 14, 21, 28} verify whether A\(B\C) = (A\B)\C.

Solution:

B\C = {6, 10, 12, 18, 24} \ {7, 10, 12, 14, 21, 28}
= {6, 18, 24}

A\(B\C) = {5, 10, 15, 20} \ {6, 18, 24}
= {5, 10, 15, 20} ----- (1)

A\B = {5, 10, 15, 20} \ {6, 10, 12, 18, 24}
= {5, 15, 20}

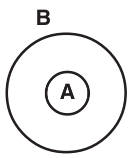
(A\B)\C = {5, 15, 20} \ {7, 10, 12, 14, 21, 28}
= {5, 15, 20} ----- (2)

(1) $\neq$ (2)

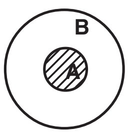
A\(B\C) $\neq$ (A\B)\C.

9. If $A \subset B$ then find $A \cap B$ and A\B. (Use Venn diagram)

Solution:



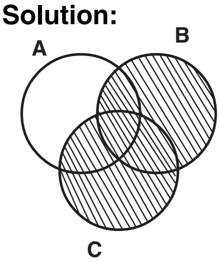
A\B = ϕ



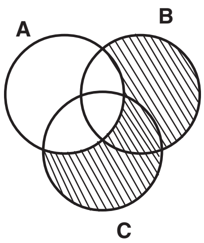
$A \cap B = A$ if $A \subset B$

10. Draw Venn diagram $(B \cup C)$\A

Solution:



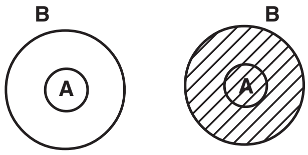
$B \cup C$



$(B \cup C)$ \ A

11. If $A \subset B$ show that $A \cup B = B$ (Use Venn diagram)

Solution:



$A \cup B = B$

12. Let X = {1, 2, 3, 4}. Examine whether the relation g = {(3,1), (4, 2), (2,1)} is a function from X to X or not Explain

Solution :

The relation

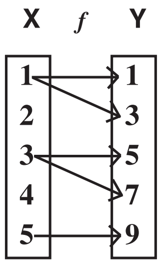
g = {(3,1), (4, 2), (2,1)} is not a function.

Reason:

Because the element 1 does not have an image. That is domain of g = {2, 3, 4} $\neq$ X.

13. X = {1, 2, 3, 4}, Y = {1, 3, 5, 7, 7, 9} determine the following relation from X to Y is a function? Give reason for your answer. If it is a function. State its type. {(1,1), (1, 3), (3, 5), (3,7), (5, 7)}

Solution:



$X \rightarrow Y$ is not a function since 3 is associated with more than one element of Y. and 2 and 4 do not have images.

14. Write the pre-images of 2 and 3 in the function. f = {(12, 2), (13, 3), (15, 3), (14, 2) (17, 17)}

Solution :

Pre-image of 2 = 12 and 14

Pre-image of 3 = 13 and 15

15. f = {(1, -1) (4, 2), (9,-3), (16, -4)} is a function from A = {1, 4, 9, 16} to B = {-1, 2, -3, -4, 5}. In case of a function write down its range

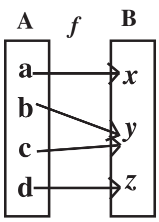
Solution :

f = {(1, -1), (4, 2), (9, -3) (16, -4)}

Each element A is associated with a unique element in B. Thus f is a function.

Range of f = {-1, 2, -3, -4}

16. State the following arrow diagram defines a function or not. Justify your answer.



Solution:

f is a function from A to B since every element in A has a unique image in B.

17. Let A = {1, 2, 3, 4, 5}, B = N and f : $A \rightarrow B$ be defined by $f(x) = x^2$. Find the range of f. Identify the type of function.

Solution:

A = {1, 2, 3, 4, 5}

$B = \{1, 2, 3, 4, ...\}$

$f(x) = x^2$

$f(1) = 1^2 = 1$

$f(2) = 2^2 = 4$

$f(3) = 3^2 = 9$

$f(4) = 4^2 = 16$

$f(5) = 5^2 = 25$

Range of f = {1, 4, 9, 16, 25}

Since distinct elements are mapped into distinct images, it is a one-one function.

18. f = {(1, 2), (4, 5), (9,-4), (16, 5)} is a relation from A = {1, 3, 9, 16} to B = {-1, 2, -3, -4, 5, 6}. In case of a function write down its range

Solution:

f = {(1, 2), (4, 5), (9,-4), (16, 5)}

Each element in A is associated with a unique element in B. Hence it is a function.

Range of f = {2, 5, -4}

2. SEQUENCES AND SERIES OF REAL NUMBERS

1. Find the 10th term and common ratio of the geometric sequences $\frac{1}{4}, \frac{-1}{2}$, 1, -2, ...

$a = \frac{1}{4}$; $r = \frac{t_2}{t_1} = \frac{-1/2}{1/4} = -1/2 \times 4 = -2$

$t_n = ar^{n-1}$

$t_{10} = \frac{1}{4}(-2)^{10-1}$

$= \frac{1}{4}(-2)^9 = (-2)^7$

$t_{10} = (-2)^7$

2. If the n^{th} term of a series is $2n^2 - 3n + 1$ find 7^{th} term.

$a_n = 2n^2 - 3n + 1$

$a_7 = 2(7)^2 - 3(7) + 1$

$= 2 \times 49 - 21 + 1$

$= 98 - 20$

$a_7 = 78$

3. Find 15th term of a series 125, 120, 115, 110.....

a = 125; d = 120 - 125 = -5

$t_n = a + (n-1)d$

$t_{15} = 125 = (15-1)(-5)$

$= 125 + 14(-5)$

$= 125 - 70$

$t_{15} = 55$

4. Find the 17^{th} term of the A.P. 4, 9, 14 ......

a = 4; d = 9 - 4 = 5

$t_n = a + (n - 1)d$

$t_{17} = 4 + (17 - 1)(5)$

$= 4 + 16(5)$

$= 4 + 80$

$t_{17} = 84$

5. Find the first term and common difference of an A.P. $\frac{1}{2}, \frac{5}{6}, \frac{7}{6}, \frac{3}{2} \ldots \frac{17}{6}$

$$a = \frac{1}{2}; \quad d = \frac{5}{6} - \frac{1}{2} = \frac{5-3}{6} = \frac{2}{6} = \frac{1}{3}$$

First term $a = \frac{1}{2}$

Common difference $d = \frac{1}{3}$

6. Three numbers are in the ratio 2 : 5 : 7. If the first number, the resulting number on the subtraction of 7 from the second number and the third number form an arithmetic sequence then find the numbers.

Let the three numbrs $2x, 5x, 7x$

If $2x, 5x - 7, 7x$ are in A.P.

$$t_2 - t_1 = t_3 - t_2$$
$$(5x - 7) - 2x = 7x - (5x - 7)$$
$$3x - 7 = 2x + 7$$
$$3x - 2x = 7 + 7$$
$$x = 14$$

Three numbers are $2x, 5x, 7x$.

$= 2 \times 14, 5 \times 14, 7 \times 14$

$= 28, 70, 98$

7. The fifth term of a G.P. is 1875. If the first term is 3, find the common ratio.

$a = 3; t_n = ar^{n-1}$

$t_5 = 1875 \Rightarrow (3)(r^4) = 1875$

$$r_4 = \frac{1875}{3}$$
$$= 625$$
$$r^4 = 5^4$$
$$r = 5$$
$$r = 5$$

8. Which term of the geometric sequence 1, 2, 4, 8..... is 1024?

$a = 1; \ r = 2/1 = 2 ; t_n = ar^{n-1}$

$t_n = 1024 \Rightarrow (1)(2)^{n-1} = 1024$

$$2^n \times 2^{-1} = 1024$$
$$2^n \times \frac{1}{2} = 2^{10}$$
$$2^n = 2^{10} \times 2^1$$
$$2^n = 2^{11}$$
$$n = 11$$

9. If a, b, c are in A.P. then prove that $(a - c)^2 = 4(b^2 - ac)$.

If a, b, c are in A.P.

$$t_2 - t_1 = t_3 - t_2$$
$$b - a = c - b$$
$$b + b = c + a$$
$$2b = c + a$$

Squaring both sides

$$4b^2 = (c+a)^2$$
$$4b^2 = a^2 + 2ac + c^2$$

Add (- 4ac) on both sides.

$$a^2 + 2ac + c^2 - 4ac = 4b^2 - 4ac$$
$$a^2 - 2ac + c^2 = 4(b^2 - ac)$$
$$(a - c)^2 = 4(b^2 - ac)$$

10. If the sum of A.P. is 1275 and first term a = 3, common difference d = 4 then find the value of n.

$S_n = 1275$

$$\frac{n}{2}[2a + (n-1)d] = 1275$$

$$\frac{n}{2}[2(3) + (n-1)4] = 1275$$

$$\frac{n}{2}[6 + 4n - 4] = 1275$$

$$\frac{n}{2}[2 + 4n] = 1275$$

$$\frac{n}{2} \times 2[1+2n] = 1275$$

$$n[1+2n] = 1275$$

$$2n^2 + n - 1275 = 0$$

$$(n - 25)(2n + 51) = 0$$

$n - 25 = 0$ (or) $2n + 51 = 0$

$n = 25$ $\qquad 2n = -51$

$\qquad n = -51/2$ (No negative value)

$\therefore \quad n = 25$

11. How many two digit numbers are divisible by 13?

Two digits numbers are 10, 11, 12, 13 99

The numbers which are divisible by 13.

= 13, 26 91

a = 13; d = 26 - 13 = 13; l = 91

$$n = \left(\frac{\ell - a}{d}\right) + 1$$

$$= \left(\frac{91 - 13}{13}\right) + 1$$

$$= \left(\frac{78}{13}\right) + 1 = 6 + 1$$

$$n = 7$$

12. In a flower garden, there are 23 rose plants in the first row, 21 in the second row, 19 in the third row and so on. There are 5 rose plants in the last row. How many rows are there in the flower garden?

23, 21, 19, ..., 5 are in A.P.

a = 23; d = 21 - 23 = -2; l = 5

$$n = \left(\frac{\ell - a}{d}\right) + 1$$

$$= \left(\frac{5 - 23}{-2}\right) + 1$$

$$= \left(\frac{-18}{-2}\right) + 1$$

$= 9 + 1$

$= 10$

There are 10 rows in the flower garden.

13. If a person joins his work in 2010 with an annual salary of Rs.30,000 and receives and annual increment of Rs.600 every year, in which year, will his annual salary be Rs.39,000?

30000, 30600, ... , 39000 are in A.P.

$\div 100 \Rightarrow$ 300, 306, ... 390 are also in A.P.

$a = 300; \ell = 390, d = 306 - 300 = 6$

$$n = \left(\frac{\ell - a}{d}\right) + 1$$

$$= \left(\frac{390 - 300}{6}\right) + 1$$

$$= \left(\frac{90}{6}\right) + 1$$

$= 15 + 1$

$n = 16$

16th annual salary of the person will be Rs.39000.

His annual salary will reach Rs.39000 in the year 2025.

14. Find the 12[th] term of the A.P. $\sqrt{2}, 3\sqrt{2}, 5\sqrt{2}$ .....

$a = \sqrt{2}$, $d = 3\sqrt{2} - \sqrt{2} = 2\sqrt{2}$, $n = 12$

$t_n = a + (n - 1) d$

$t_{12} = \sqrt{2} + (12-1)\ 2\sqrt{2}$

$= \sqrt{2} + 24\sqrt{2} - 2\sqrt{2}$

$t_{12} = 23\sqrt{2}$

15. Find the common ratio and the general term of the series $\frac{2}{5}, \frac{6}{25}, \frac{18}{125}$....

$$a = \frac{2}{5}; \quad r = {}^{6/25}\!\!\diagup_{2/5} = \frac{6}{25} \times \frac{5}{2} = \frac{3}{5}$$

Common ratio, $r = \frac{3}{5}$

Common term, $t_n = ar^{n-1}$

$$= \left(\frac{2}{5}\right)\left(\frac{3}{5}\right)^{n-1}, n = 1, 2, 3 .....$$

16. Find the common ratio and general term of the series 0.02, 0.006, 0.0018.....

$$a = 0.02, \; r = \frac{0.006}{0.02} = 0.3 = \frac{3}{10}$$

Common ratio, $r = \frac{3}{10}$

Common term, $t_n = ar^{n-1}$

$$= (0.02)\left(\frac{3}{10}\right)^{n-1}, n = 1, 2, 3....$$

17. Find the sum first 125 natural numbers.

$$\sum n = \frac{n(n+1)}{2}$$

$$1 + 2 + ... + 125 = \frac{125 \times 126}{2}$$

$= 125 \times 63$

$= 7875$

18. Find the sum of 75 positive integers.

$$\sum n = \frac{n(n+1)}{2}$$

$$1 + 2 + ... + 75 = \frac{75 \times 76}{2}$$

$= 75 \times 38$

$= 2850$

19. Find the sum of the series 1 + 3 + 5 + ... , 25 terms.

$\sum 2n - 1 = n^2$

$1 + 3 + 5 + ... , 25 \text{ terms} = 25^2$

$= 625$

20. Find the sum of the series 31 + 33 + ... + 53

$$\sum 2n - 1 = \left(\frac{\ell + 1}{2}\right)^2$$

$31 + 33 + ... + 53 = (1 + 3 + ... + 53) - (1 + 3 + ... + 29)$

$$= \left[\frac{53+1}{2}\right]^2 - \left[\frac{29+1}{2}\right]^2$$

$$= \left(\frac{54}{2}\right)^2 - \left(\frac{30}{2}\right)^2$$

$= 27^2 - 15^2$

$= (27 + 15)(27 - 15)$

$= 42 \times 12$

$= 504.$

21. Find the sum of the series $1^3 + 2^3 + 3^3 + ... + 20^3$

$$\sum n^3 = \left[\frac{n(n+1)}{2}\right]^2$$

$$1^3 + 2^3 + 3^3 + ... + 20^3 = \left[\frac{20 \times 21}{2}\right]^2$$

$= [10 \times 21]^2$

$= (210)^2$

$= 44100$

22. If $1^3 + 2^3 + 3^3 + ... + n^3 = 36100$ find the value of 1 + 2 + 3 + ... + n

$\sum n^3 = [\sum n]^2$

$1^3 + 2^3 + 3^3 + ... + n^3 = 36100$

$\sum n^3 = 36100$

$[\sum n]^2 = 36100 \quad [\because \sum n^3 = [\sum n]^2]$

$\sum n = \sqrt{36100}$

$= \sqrt{19 \times 19} \quad = 19$

$1 + 2 + ... + n \quad = 19$

23. Find the sum of the series 2 + 4+ 6 + ...+100

2 + 4 + 6 + ... + 100 = 2 (1 + 2 + 3 + ... +50)

$$= 2\left(\frac{50\times51}{2}\right) \qquad \left[\because \sum n = \frac{n(n+1)}{2}\right]$$

= 50 x 51

= 2550

24. Find the sum of the series 7 + 14 + 21 + ... + 490

7 + 14 + 21 + ... + 490 = 7 [1 + 2 + 3 + ... + 70]

$$= 7\left(\frac{70\times71}{2}\right) \qquad \left[\because \sum n = \frac{n(n+1)}{2}\right]$$

= 7 x 35 x 7

= 17395

25. A gardener plans to construct a trapezoidal shaped structure in his garden. The longer side of trapezoid need to start with a row of 97 bricks. Each row must be decreased by 2 bricks on each end and the construction should stop at 25th row. How many bricks does he need to buy?

97 + 93 + 89 + ... 25 terms

a = 97; d = -4; n = 25

$$S_n = \frac{n}{2}[\,2a + (n-1)d]$$

$$S_{25} = \frac{25}{2}[2\,(97) + (24)\,(-4)]$$

$$= \frac{25}{2}(194 - 96)$$

$$= \frac{25}{2} \times 98$$

= 1225

1225 bricks are need.

26. If a clock strickes once at 1 o'clock, twice at 2 o'clock and so on, how many times will it strike in a day.

The number of times the clock strike in a day = 2 (1+2 +.... 12)

$$= 2\left(\frac{12\times13}{2}\right)$$

= 156

3. ALGEBRA

1. Find a quadratic polynomial if the sum and product of zeros of it are -4 and 3 respectively.

Let α, β be the zeros.

Sum of zeros = $\alpha + \beta = -4$

Product of zeros = $\alpha\beta = 3$

The polynomial is $P(x) = x^2 - (\alpha+\beta)\,x + \alpha\beta$

$= x^2 - (-4)\,x + 3$

$= x^2 + 4x + 3$

2. Find the quotient and remainder when $x^3 + x^2 - 7x - 3$ is divided by $x - 3$

3	1	1	-7	-3
	0	3	12	15
	1	4	5	12

Quotient $= x^2 + 4x + 5$

Remainder = 12

3. Prove that $x - 1$ is a factor of $x^3 - 6x^2 + 11x - 6$

$P(x) = x^3 - 6x^2 + 11x - 6$

$P(x) = (1)^3 - 6(1)^2 + 11(1) - 6$

$= 1 - 6(1) + 11 - 6$

$= 1 - 6 + 11 - 6$

$= 12 - 12$

$= 0$

$\therefore (x-1)$ is a factor

4. Find the G.C.D. of x^2y, x^3y, x^2y^2

G.C.D. $= x^2y$

5. Find the L.C.M. of i) a^2bc, b^2ca, c^2ab, ii) a^{m+1}, a^{m+2}, a^{m+3}

i) L.C.M. $= a^2b^2c^2$

ii) L.C.M. $= a^{m+3}$

6. Simplify : $\frac{5x+20}{7x+28}$

$$\frac{5x+20}{7x+28} = \frac{5(x+4)}{7(x+4)}$$

$$= \frac{5}{7}$$

7. Find the square root of

i) $\frac{81x^4y^6z^8}{64W^{12}S^{14}}$ ii) $121(x - a)^4 (x - b)^6 (x - c)^{12}$

i) Square root $= \left|\frac{9x^2y^3z^4}{8W^6S^7}\right|$

ii) Square root $= |11(x - a)^2 (x - b)^3 (x - c)^6|$

8. Determine the nature of the roots i) $x^2 - 11x - 10 = 0$ ii) $9x^2+12x+4=0$

i) Discriminant is $\Delta = b^2 - 4ac$

a = 1, b = -11, c = -10 $= (-11)^2 - 4(1)(-10)$

$= 121 + 40$

$= 161$

$\Delta > 0$. Roots are real, unequal.

ii) Discriminant is $\Delta = b^2 - 4ac$

a = 9, b = 12, c = 4

$= (12)^2 - 4(9)(4)$

$= 144 - 144 = 0$

$\Delta = 0$ Roots are real and equal.

9. If the roots of $2x^2 - 10x + k = 0$ are real and equal, find the value of k.

Given that the roots are equal

$$b^2 - 4ac = 0$$

a = 2, b = -10, c = k

$$(-10)^2 - 4(2)(k) = 0$$

$$100 - 8k = 0$$

$$-8k = -100$$

$$k = \frac{100}{8}$$

$$k = \frac{25}{2}$$

10. Find the quadratic equation whose roots are $7+\sqrt{3}$ and $7-\sqrt{3}$

Given $7+\sqrt{3}$ and $7-\sqrt{3}$ are the roots.

Sum of the roots = $7+\sqrt{3}$ + $7-\sqrt{3}$ = 14

Product of the roots = $(7+\sqrt{3})(7-\sqrt{3})$ = 49 - 3 = 46

Required equation

x^2 - (S.O.R) x + P.O.R. = 0

$x^2 - 14x + 46 = 0$

4. MATRICES

1. Let A = $(a_{ij}) = \begin{pmatrix} 1 & 4 & 8 \\ 6 & 2 & 5 \\ 3 & 7 & 0 \\ 9 & -2 & -1 \end{pmatrix}$ find (i) the order of the matrix.

ii) the elements a_{13} and a_{42} iii) the position of the element 2.

Solution :

i) A is of order = 4 x 3

ii) the element $a_{13} = 8$

$a_{42} = -2$

iii) the position of the element 2 is = a_{22}

2. Construct a 2 x 3 matrix whose elements are given by $a_{ij} = |2i - 3j|$.

Solution:

The general form of 2 x 3 matrix = $\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{pmatrix}$

$a_{11} = |2(1) - 3(1)| = |2 - 3| = |-1| = 1$

$a_{12} = |2(1) - 3(2)| = |2 - 6| = |-4| = 4$

$a_{13} = |2(1) - 3(3)| = |2 - 9| = |-7| = 7$

$a_{21} = |2(2) - 3(1)| = |4 - 3| = |1| = 1$

$a_{22} = |2(2) - 3(2)| = |4 - 6| = |-2| = 2$

$a_{23} = |2(2) - 3(3)| = |4 - 9| = |-5| = 5$

Hence the required matrix A = $\begin{pmatrix} 1 & 4 & 7 \\ 1 & 2 & 5 \end{pmatrix}$

3. Construct a 2 x 2 matrix $A = [a_{ij}]$ whose elements are given by

i) $a_{ij} = ij$

$a_{11} = 1 \times 1 = 1$ $\quad a_{12} = 1 \times 2 = 2$ $\quad \therefore A = \begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}$

$a_{21} = 2 \times 1 = 2$ $\quad a_{22} = 2 \times 2 = 4$

ii) $a_{ij} = 2i - j$

$a_{11} = 2(1) - 1 = 2 - 1 = 1$

$a_{12} = 2(1) - 2 = 2 - 2 = 0$

$a_{21} = 2(2) - 1 = 4 - 1 = 3$

$a_{22} = 2(2) - 2 = 4 - 2 = 2$

$$A = \begin{pmatrix} 1 & 0 \\ 3 & 2 \end{pmatrix}$$

iii) $a_{ij} = \frac{i-j}{i+j}$

$a_{11} = \frac{1-1}{1+1} = \frac{0}{2} = 0$ $\qquad a_{12} = \frac{1-2}{1+2} = \frac{-1}{3}$

$a_{21} = \frac{2-1}{2+1} = \frac{1}{3}$ $\qquad a_{22} = \frac{2-2}{2+2} = \frac{0}{4} = 0$

$$A = \begin{pmatrix} 0 & -1/3 \\ 1/3 & 0 \end{pmatrix}$$

4. If $A = \begin{pmatrix} 8 & 5 & 2 \\ 1 & -3 & 4 \end{pmatrix}$ then find A^T and $(A^T)^T$.

Solution :

$$A = \begin{pmatrix} 8 & 5 & 2 \\ 1 & -3 & 4 \end{pmatrix}$$

$$A^T = \begin{pmatrix} 8 & 1 \\ 5 & -3 \\ 2 & 4 \end{pmatrix}$$

$$(A^T)^T = \begin{pmatrix} 8 & 5 & 2 \\ 1 & -3 & 4 \end{pmatrix}$$

5. If $A = \begin{pmatrix} 1 & -1 & 3 & 2 \\ 5 & -4 & 7 & 4 \\ 6 & 0 & 9 & 8 \end{pmatrix}$ i) Find the order of the matrix ii) Write down the elements a_{24} and a_{32}

iii) in which row and column does the element 7 occur?

Solution :

i) 3 x 4

ii) $a_{24} = 4$ $\quad a_{32} = 0$

iii) $a_{23} = 7$

6. Find the order of the following matrices:

i) $\begin{pmatrix} 1 & -1 & 5 \\ -2 & 3 & 4 \end{pmatrix}$ order of the matrix = 2 x 3

ii) $\begin{pmatrix} 7 \\ 8 \\ 9 \end{pmatrix}$ order of the martrix = 3 x 1

iii) $\begin{pmatrix} 3 & -2 & 6 \\ 6 & -1 & 1 \\ 2 & 4 & 5 \end{pmatrix}$ order of the matrix = 3 x 3

iv) $(3 \quad 4 \quad 5)$ order of the matrix = 1 x 3

v) $\begin{pmatrix} 1 & 2 \\ -2 & 3 \\ 9 & 7 \\ 6 & 4 \end{pmatrix}$ order of the matrix = 4 x 2

7. A matrix consists of 30 elements. What are the possible orders it can have?

1 x 30
30 x 1
2 x 15
15 x 2
3 x 10
10 x 3
5 x 6
6 x 5

Row	1	2	3	5
Column	30	15	10	6

8. If A= $\begin{pmatrix} 1 & 3 \\ 4 & 1 \\ 5 & 0 \end{pmatrix}$ then find the transpose of A.

$$A^T = \begin{pmatrix} 1 & 4 & 5 \\ 3 & 1 & 0 \end{pmatrix}$$

9. If $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & -5 \\ 3 & -5 & 6 \end{pmatrix}$ then verify that $(A^T)^T = A$

$$A^T = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & -5 \\ 3 & -5 & 6 \end{pmatrix} \Rightarrow (A^T)^T = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & -5 \\ 3 & -5 & 6 \end{pmatrix}$$

$(A^T)^T = A$

10. Find the values of x, y and z if $\begin{pmatrix} x & 5 & 4 \\ 5 & 9 & 1 \end{pmatrix} = \begin{pmatrix} 3 & 5 & z \\ 5 & y & 1 \end{pmatrix}$

X = 3, Y = 9, Z = 4

11. If $A = \begin{pmatrix} -1 & 2 & 4 \\ 3 & 6 & -5 \end{pmatrix}$ then find 3A.

$$3A = 3\begin{pmatrix} -1 & 2 & 4 \\ 3 & 6 & -5 \end{pmatrix}$$

$$= \begin{pmatrix} 3(-1) & 3(2) & 3(4) \\ 3(3) & 3(6) & 3(-5) \end{pmatrix}$$

$$= \begin{pmatrix} -3 & 6 & 12 \\ 9 & 18 & -15 \end{pmatrix}$$

12. If $A = \begin{pmatrix} 5 & 6 & -2 & 3 \\ 1 & 0 & 4 & 2 \end{pmatrix}$, $B = \begin{pmatrix} 3 & -1 & 4 & 7 \\ 2 & 8 & 2 & 3 \end{pmatrix}$ then find that A + B.

Solution:

$$A + B = \begin{pmatrix} 5 & 6 & -2 & 3 \\ 1 & 0 & 4 & 2 \end{pmatrix} + \begin{pmatrix} 3 & -1 & 4 & 7 \\ 2 & 8 & 2 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 5+3 & 6-1 & -2+4 & 3+7 \\ 1+2 & 0+8 & 4+2 & 2+3 \end{pmatrix}$$

$$A + B = \begin{pmatrix} 8 & 5 & 2 & 10 \\ 3 & 8 & 6 & 5 \end{pmatrix}$$

13. If $A = \begin{pmatrix} 2 & 3 \\ -9 & 5 \end{pmatrix} - \begin{pmatrix} 1 & 5 \\ 7 & -1 \end{pmatrix}$, then find the additive inverse of A.

Solution :

$$A = \begin{pmatrix} 2 & 3 \\ -9 & 5 \end{pmatrix} - \begin{pmatrix} 1 & 5 \\ 7 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} 2-1 & 3-5 \\ -9-7 & 5-(-)1 \end{pmatrix} = \begin{pmatrix} 1 & -2 \\ -16 & 6 \end{pmatrix}$$

$$\text{Additive inverse of A} = \begin{pmatrix} -1 & 2 \\ 16 & -6 \end{pmatrix}$$

14. Let $A = \begin{pmatrix} 3 & 2 \\ 5 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 8 & -1 \\ 4 & 3 \end{pmatrix}$. Find the matrix C = 2A + B.

Solution :

$$C = 2\begin{pmatrix} 3 & 2 \\ 5 & 1 \end{pmatrix} + \begin{pmatrix} 8 & -1 \\ 4 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 6 & 4 \\ 10 & 2 \end{pmatrix} + \begin{pmatrix} 8 & -1 \\ 4 & 3 \end{pmatrix} = \begin{pmatrix} 6+8 & 4-1 \\ 10+4 & 2+3 \end{pmatrix}$$

$$C = \begin{pmatrix} 14 & 3 \\ 14 & 5 \end{pmatrix}$$

15. If $A = \begin{pmatrix} 4 & -2 \\ 5 & -9 \end{pmatrix}$ and $B = \begin{pmatrix} 8 & 2 \\ -1 & -3 \end{pmatrix}$ find 6A - 3B.

Solution:

$$6A - 3B = 6\begin{pmatrix} 4 & -2 \\ 5 & -9 \end{pmatrix} - 3\begin{pmatrix} 8 & 2 \\ -1 & -3 \end{pmatrix}$$

$$= \begin{pmatrix} 6\times4 & 6\times-2 \\ 6\times5 & 6\times-9 \end{pmatrix} - \begin{pmatrix} 3\times8 & 3\times2 \\ 3\times-1 & 3\times-3 \end{pmatrix}$$

$$= \begin{pmatrix} 24 & -12 \\ 30 & -54 \end{pmatrix} + \begin{pmatrix} 24 & 6 \\ -3 & -9 \end{pmatrix}$$

$$= \begin{pmatrix} 24-24 & -12-6 \\ 30+3 & -54+9 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & -18 \\ 33 & -45 \end{pmatrix}$$

16. If $A = \begin{pmatrix} 1 & 3 \\ 9 & -6 \end{pmatrix}$ then verify $AI = IA = A$ where I is the unit matrix of order 2.

Solution :

$$AI = \begin{pmatrix} 1 & 3 \\ 9 & -6 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1+0 & 0+3 \\ 9+0 & 0-6 \end{pmatrix} \quad \begin{matrix} (1\ \ 3)\begin{pmatrix} 1 \\ 0 \end{pmatrix} & (1\ \ 3)\begin{pmatrix} 0 \\ 1 \end{pmatrix} \\ (9\ -6)\begin{pmatrix} 1 \\ 0 \end{pmatrix} & (9\ -6)\begin{pmatrix} 0 \\ 1 \end{pmatrix} \end{matrix}$$

$$= \begin{pmatrix} 1 & 3 \\ 9 & -6 \end{pmatrix}$$

$AI = A$

$$IA = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 3 \\ 9 & -6 \end{pmatrix}$$

$$= \begin{pmatrix} 1+0 & 3+0 \\ 0+9 & 0-6 \end{pmatrix} \quad \begin{matrix} (1\ \ 0)\begin{pmatrix} 1 \\ 9 \end{pmatrix} & (1\ \ 0)\begin{pmatrix} 3 \\ -6 \end{pmatrix} \\ (0\ \ 1)\begin{pmatrix} 1 \\ 9 \end{pmatrix} & (0\ \ 1)\begin{pmatrix} 3 \\ -6 \end{pmatrix} \end{matrix}$$

$$= \begin{pmatrix} 1 & 3 \\ 9 & -6 \end{pmatrix}$$

$IA = A$

$\therefore AI = IA = A$. Hence proved.

17. Find the product of the matrices, if exists

i) $(2\ \ -1)\begin{pmatrix} 5 \\ 4 \end{pmatrix} = (10 - 4) = (6)$

ii) $\begin{pmatrix} 3 & -2 \\ 5 & 1 \end{pmatrix} \begin{pmatrix} 4 & 1 \\ 2 & 7 \end{pmatrix}$

$$= \begin{pmatrix} 12-4 & 3-14 \\ 20+2 & 5+7 \end{pmatrix} \quad \begin{matrix} (3\ -2)\begin{pmatrix} 4 \\ 2 \end{pmatrix} & (3\ -2)\begin{pmatrix} 1 \\ 7 \end{pmatrix} \\ (5\ \ 1)\begin{pmatrix} 4 \\ 2 \end{pmatrix} & (5\ \ 1)\begin{pmatrix} 1 \\ 7 \end{pmatrix} \end{matrix}$$

$$= \begin{pmatrix} 8 & -11 \\ 22 & 12 \end{pmatrix}$$

iii) $\begin{pmatrix} 2 & 9 & -3 \\ 4 & -1 & 0 \end{pmatrix} \begin{pmatrix} 4 & 2 \\ -6 & 7 \\ -2 & 1 \end{pmatrix}$

$$= \begin{pmatrix} 8-54+6 & 4+63-3 \\ 16+6+0 & 8-7+0 \end{pmatrix} \qquad (2\ \ 9\ \ -3)\begin{pmatrix} 4 \\ -6 \\ -2 \end{pmatrix} \quad (2\ \ 9\ \ -3)\begin{pmatrix} 2 \\ 7 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} -40 & 64 \\ 22 & 1 \end{pmatrix}$$

iv) $\begin{pmatrix} 6 \\ -3 \end{pmatrix}(2\ \ -7) = \begin{pmatrix} 12 & -42 \\ -6 & 21 \end{pmatrix}$

18. If A = $\begin{pmatrix} 8 & -7 \\ -2 & 4 \\ 0 & 3 \end{pmatrix}$ and B = $\begin{pmatrix} 9 & -3 & 2 \\ 6 & -1 & -5 \end{pmatrix}$ then find AB and BA.

Solution:

$$AB = \begin{pmatrix} 8 & -7 \\ -2 & 4 \\ 0 & 3 \end{pmatrix} \begin{pmatrix} 9 & -3 & 2 \\ 6 & -1 & -5 \end{pmatrix}$$

$$= \begin{pmatrix} 72-42 & -24+7 & 16+35 \\ -18+24 & 6-4 & -4-20 \\ 0+18 & 0-3 & 0-15 \end{pmatrix}$$

$$= \begin{pmatrix} 30 & -17 & 51 \\ 6 & 2 & -24 \\ 18 & -3 & -15 \end{pmatrix}$$

$$BA = \begin{pmatrix} 9 & -3 & 2 \\ 6 & -1 & -5 \end{pmatrix} \begin{pmatrix} 8 & -7 \\ -2 & 4 \\ 0 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 72+6+0 & -63-12+6 \\ 48+2+0 & -42-4-15 \end{pmatrix}$$

$$BA = \begin{pmatrix} 78 & -69 \\ 50 & -61 \end{pmatrix}$$

5. COORDINATE GEOMETRY

1. Find the midpoint of the line segment joining the points (3, 0) and (-1, 4)

Midpoint M (x, y) of the line segment joining the points (x_1, y_1) and (x_2, y_2) is

$$M(x,y) = M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Midpoint of the line segment joining the points (3, 0) and (-1, 4) is

$$M(x,y) = \left(\frac{3-1}{2}, \frac{0+4}{2}\right) = M\ (1, 2)$$

2. In what ratio does the point P (-2, 3) divide the line segment joining the points A (-3, 5) and B (4, -9) internally?

Given points are A (-3, 5) and B (4, -9).

Let P (-2, 3) divides AB internally in the ratio ℓ : m.

By the section formula

$$P\left(\frac{\ell x_2 + m x_1}{\ell + m}, \frac{\ell y_2 + m y_1}{\ell + m}\right) = P\ (-2, 3)$$

$x_1 = -3,\ y_1 = 5,\ x_2 = 4,\ y_2 = -9$

$$\left(\frac{4\ell - 3m}{\ell + m}, \frac{-9\ell + 5m}{\ell + m}\right) = (-2, 3)$$

Equating the x-ordinates we get $\frac{4\ell - 3m}{\ell + m} = -2$

$6l = m$

$\frac{\ell}{m} = \frac{1}{6}$ $\quad \ell : m = 1 : 6$

Hence P divides AB internally in the ratio 1 : 6

3. Find the centroid of the triangle whose vertices are A (4, -6) B (3, -2) and C (5, 2)

The centroid of a triangle is

$$G\ (x, y) = G\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

We have $(x_1, y_1) = (4, -6)$

$(x_2, y_2) = (3, -2)$

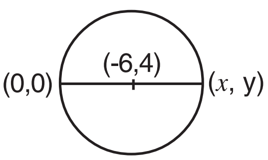
$(x_3, y_3) = (5, 2)$

The centroid of the triangle whose vertices are (4, -6), (3, -2), (5, 2) is

$$G\ (x, y) = G\left(\frac{4+3+5}{3}\ \frac{-6-2+2}{3}\right) = G\ (4, -2)$$

4. The centre of a circle is at (-6, 4). If one end of a diameter of the circle is at the origin, then find the other end. **(June 12)**

Solution:



AB is the diameter, O is the centre

Mid point of AB = $\left(\frac{0+x}{2}, \frac{0+y}{2}\right) = (-6, 4)$

$\frac{x}{2} = -6 \quad \Rightarrow \quad x = -12$

$\frac{y}{2} = 4 \quad \Rightarrow \quad y = 8$

The other end of the diameter is (-12, 8)

5. If the centroid of a triangle is at (1, 3) and two of its vertices are (-7, 6) and (8, 5) then find the third vertex of the triangle . **(Apr. 12)**

Solution :

Given centroid is (1, 3) and two vertices of a triangle are (-7, 6), (8, 5) the 3rd vertex (x, y)

$$\left(\frac{-7+8+x}{3},\frac{6+5+y}{3}\right) = (1, 3)$$

$\frac{x+1}{3} = 1$ and $\frac{y+11}{3} = 3$

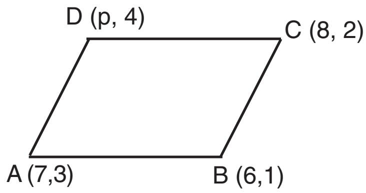
$x = 2$ $y = -2$

The third vertex is (2, -2).

6. If (7,3) (6,1) (8,2) and (p, 4) are the vertices of a parallelogram taken in order, then find the value of p.

Solution :

Diagonals of a parallelogram bisect each other



Hence $\left(\frac{7+8}{2},\frac{3+2}{2}\right) = \left(\frac{6+p}{2},\frac{1+4}{2}\right)$

$$\left(\frac{6+p}{2},\frac{5}{2}\right) = \left(\frac{15}{2},\frac{5}{2}\right)$$

Equating x-coordinates

$$\frac{6+p}{2} = \frac{15}{2}$$

$p = 9$

7. Find the coordinates of the point which divides the line segment joining (3, 4) and (–6, 2) in the ratio 3 : 2 externally.

Solution :

A (3, 4), B (-6, 2) be the given points.

P(x, y) divides AB externally in the ratio 3 : 2.

By section formula $\ell = 3$ $x_1 = 3$ $x_2 = -6$

m = 2 $y_1 = 4$ $y_2 = 2$

$$\left(\frac{\ell x_2 - m x_1}{\ell - m},\frac{\ell y_2 - m y_1}{\ell - m}\right) = (x, y)$$

$$(x, y) = \left(\frac{-18-6}{1}, \frac{6-8}{1}\right)$$

$(x,y) = (-24, -2)$

8. Find the coordinates of the point which divides the line segment joining (-3, 5) and (4, -9) in the ratio 1 : 6 internally. **(Sep. 14)** ★

Solution:

A (-3, 5) B (4, -9) be the given points P(x, y) divides AB internally in the ratio 1 : 6.

By section formula

$$\left(\frac{\ell x_2 + m x_1}{\ell + m},\frac{\ell y_2 + m y_1}{\ell + m}\right) = (x, y)$$

$x_1 = -3$ $x_2 = 4$ $\ell = 1$, m = 6

$$(x, y) = \left(\frac{(1\times4)+6(-3)}{1+6},\frac{1(-9)+(6\times5)}{1+6}\right)$$

$y_1 = 5$ $y_2 = -9$

$$(x,y) = \left(\frac{-14}{7},\frac{21}{7}\right) = (-2, 3)$$

9. If the area of the ΔABC is 68 sq. units and the vertices are A (6,7), B (-4, 1) and C (a, -9) taken in order, then find the value of 'a'.

Solution:

Area of ΔABC is $= \frac{1}{2}\begin{vmatrix} 6 & -4 & a & 6 \\ 7 & 1 & -9 & 7 \end{vmatrix}$

$$\Delta = \frac{1}{2}[(6+36+7a) - (-28 + a - 54)] = 68$$

$$(42+7a) - (a - 82) = 136$$

$$6a = 12$$

$$a = 2$$

10. Show that the points A (2, 3), B(4,0) and C(6, -3) are collinear.

Area of the ΔABC $= \frac{1}{2}\begin{vmatrix} 2 & 4 & 6 & 2 \\ 3 & 0 & -3 & 3 \end{vmatrix}$

$$\Delta = \frac{1}{2}[(0 - 12 + 18) - (12 + 0 - 6)]$$

$$= \frac{1}{2}[6 - 6]$$

$$= 0$$

The given points are collinear.

11. Find the angle of inclinatiion of the straight line whose slope is $\frac{1}{\sqrt{3}}$.

If θ is the angle of inclination of the line then the slope of the line is m = tanθ.

$$\tan\theta = \frac{1}{\sqrt{3}} \Rightarrow \theta = 30^\circ$$

12. Find the slope of the straight line whose angle of inclination is 45°.

If θ is the angle of inclination of the line then the slope of the line is m = tanθ.

m = tan45°

m = 1

13. The side AB of a square ABCD is parallel to x-axis find the (i) slope of AB ii) Slope of BC iii) Slope of the diagonal.

i) The side AB is parallel to x-axis slope of AB = 0

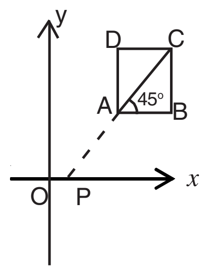
ii) The side BC is perpendicular to x axis.

Slope of BC = undefined.

ii) $\angle$BAC = 45°

$\angle$XPA = 45°

Slope of the diagonal AC = tan45° = 1



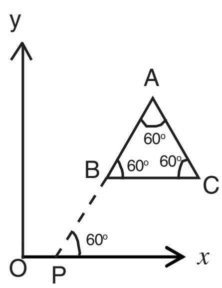
14. The side BC of an equilateral ΔABC is parallel to x-axis. Find the slope of AB and the slope of BC.

i) The side BC of ΔABC is parallel to x-axis slope of BC = 0

ii) ΔABC is equilateral $\angle$B = 60°.

$\angle$XPA = 60°

Slope of AB = tan60° = $\sqrt{3}$



15. If the points (a, 1) (1, 2) and (0, b+1) are collinear then show that $\frac{1}{a}+\frac{1}{b}$ = 1. **(Mar. 2013)**

A (a, 1) B (1, 2) C (0, b+1) are collinear.
Slope of AB = Slope of BC

$$\frac{2-1}{1-a} = \frac{b+1-2}{0-1}$$

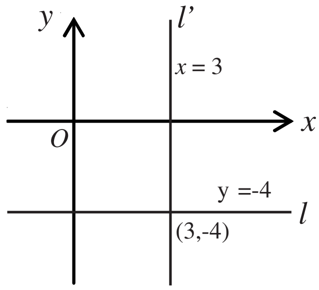
$$\frac{1}{1-a} = \frac{b-1}{-1}$$

(1-a) (b-1) = -1
(a - 1) (b -1) = 1
ab - a - b + 1 = 1
ab - a - b = 0
ab = a + b

$$\frac{a}{ab}+\frac{b}{ab} = 1$$

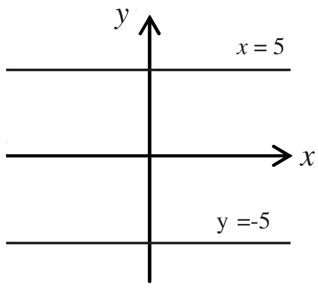
$$\frac{1}{a}+\frac{1}{b} = 1$$

16. Find the equation of the straight lines parallel to the co-ordinate axes and passing through the point (3, -4) **(April - 14)**



l and *l'* be the straight lines passing through (3, -4) and parallel to *x*-axis and y axis respectively.
The equation of the line *l* is y = -4
The equation of the line *l'* is *x* = 3.

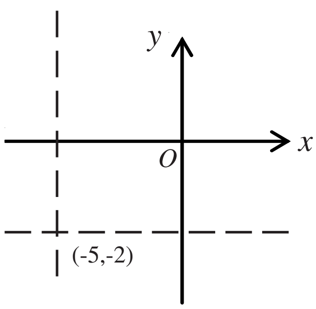
17. Write the equations of the straight lines parallel to *x* axis which are at a distance of 5 units from the *x* axis.



Equation of a straight line parallel to *x* axis at a distance of 5 units from x axis is

y = 5 or y = -5
y - 5 = 0 or y + 5 = 0

18. Find the equation of the straight lines parallel to the co-ordinate axes and passing through the point (-5,-2).



Equation of a straight line passing through (-5, -2) and

i) Parallel to *x* axis is
y = -2
y + 2 = 0

ii) Parallel to y axis is
x = -5
x + 5 = 0

19. Find the equation of the line intersecting the y axis at a distance of 3 units above the origin and $\tan\theta = 1/2$, where θ is the angle of inclination.

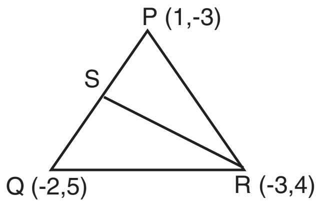
The required equation of line is $y = mx + c$

$$y = \frac{1}{2}x + 3$$

$$2y = x + 6$$

$$x - 2y + 6 = 0$$

20. Find the equation of the median from the vertex R in a ΔPQR with vertices at P (1, -3) Q (-2, 5) and R (-3, 4).



Let S be the midpoint of PQ

$$S = \left(\frac{1-2}{2}, \frac{-3+5}{2}\right)$$

$$S = \left(\frac{-1}{2}, 1\right)$$

Equation of median RS is

$$\frac{y-4}{1-4} = \frac{x-3}{-1/2+3} \Rightarrow \frac{y-4}{-3} = \frac{2(x+3)}{5}$$

$$5y - 20 = -6x - 18$$

$$6x + 5y - 20 + 18 = 0$$

$$6x + 5y - 2 = 0$$

21. Find the equation of the straight line passing through the point (3, 4) and has intercepts which are in the ratio 3 : 2.

Let the x and y intercepts of the line be a, b respectively.

$$a : b = 3 : 2$$

$$3b = 2a$$

$$b = \frac{2a}{3}$$

Equation of straight line in intercept form is

$$\frac{x}{a} + \frac{y}{2a/3} = 1 \Rightarrow \frac{x}{a} + \frac{3y}{2a} = 1$$

$$\Rightarrow \frac{2x+3y}{2a} = 1 \Rightarrow 2x + 3y = 2a \text{ passes through } (3, 4)$$

$$(2 \times 3) + (3 \times 4) = 2a \Rightarrow 2a = 18$$

$$a = 9$$

The required equation is $2x + 3y - 18 = 0$

22. Show that the straight line $3x + 2y = 12$ and $6x + 4y + 8 = 0$ are parallel.

Slope of the straight line $3x + 2y - 12 = 0$ is $m_1 = \frac{-3}{2}$

Slope of the straight line $6x + 4y + 8 = 0$ is $m_2 = \frac{-6}{4} = \frac{-3}{2}$

Since $m_1 = m_2$ then the two straight lines are parallel.

23. Prove that the straight lines $x + 2y + 1 = 0$ and $2x - y + 5 = 0$ are perpendicular to each other.

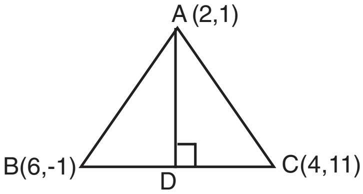
Slope of the straight line $x + 2y + 1 = 0$ is $m_1 = \frac{-1}{2}$

Slope of the straight line $2x - y + 5 = 0$ is $m_2 = \frac{-2}{-1} = 2$

Product of slopes $m_1 \times m_2 = \frac{-1}{2} \times 2 = -1$

The two straight lines are perpendicular.

24. The vertices of ΔABC are A (2, 1) B (6, -1) and C (4, 11). Find the equation of the straight line along the attitutde from the vertex A.



Slope of BC = $\frac{11+1}{4-6} = \frac{12}{-2} = -6$

Since the line AD is perpendicular to the line BC then.

Slope of AD = $\frac{1}{6}$

Equation of AD is $y - y_1 = m (x - x_1)$

$$y - 1 = \frac{1}{6} (x - 2)$$

$$6y - 6 = x - 2$$

Equation of the required straight line $x - 6y + 4 = 0$

25. Find the equation of the straight line parallel to the line $3x - y + 7 = 0$ and passing through the point (1, -2).

Equation of the straight line parallel to $3x - y + 7 = 0$ is of the form $3x - y + k = 0$ since it passes through (1, -2)

$3(1) + 2 + k = 0 \Rightarrow k = -5$

Equation of the required straight line is $3x - y - 5 = 0$

26. Find the equation of straight line whose angle of inclination is 45° and y intercept is 2/5. (Oc. 2013).

Slope of the line $m = \tan 45^\circ = 1$

y intercept $C = \frac{2}{5}$

The equation of the straight line in slope - intercept form is $y = x + \frac{2}{5} \Rightarrow y = \frac{5x+2}{5}$

$$5y = 5x + 2$$

$$5x - 5y + 2 = 0$$

27. Find the equation of the straight line which passes through the midpoint of the line segment joining (4, 2) and (3, 1) whose angle of inclination is 30°. **(Oct.12).**

Midpoint of (4, 2) and (3, 1) is $\left(\frac{7}{2}, \frac{3}{2}\right)$

Slope $m = \tan 30^\circ = \frac{1}{\sqrt{3}}$

Slope point form is $y - y_1 = m (x - x_1)$

$$y - \frac{3}{2} = \frac{1}{\sqrt{3}} \left(x - \frac{7}{2}\right)$$

$$\frac{2y-3}{2} = \frac{1}{2\sqrt{3}} (2x - 7)$$

$$(2y - 3) \sqrt{3} = (2x - 7)$$

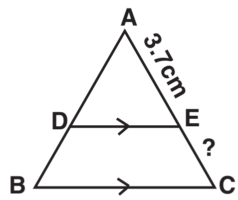
$$2x - 2\sqrt{3} y + (3\sqrt{3} - 7) = 0$$

6. GEOMETRY

1. In ΔABC, DE||BC and $\frac{AD}{DB} = \frac{2}{3}$. If AE = 3.7 cm find EC. **(June 12, June 14)**

In ΔABC, DE || BC

$\frac{AD}{DB} = \frac{AE}{EC}$ (Thales theorem0



$\frac{2}{3} = \frac{3.7}{EC}$

2 x EC = 3 x 3.7

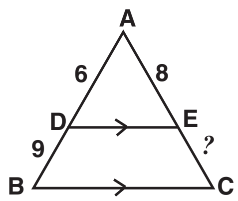
$EC = \frac{3.7 \times 3}{2} = 5.55$ cm

EC = 5.55cm

2. In a ΔABC, D and E are points on the sides AB and AC respectively such that DE || BC. If AD = 6cm, DB = 9 cm and AE = 8cm, then find AC.

In ΔABC, DE || BC.

$\therefore \frac{AD}{DB} = \frac{AE}{EC}$



$\frac{6}{9} = \frac{8}{EC}$

6 x EC = 8 x 9

$EC = \frac{8 \times 9}{6}$

EC = 12 cm

AC = AE + EC

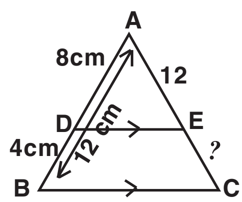
= 8 + 12 = 20

= 20 cm

3. In a ΔABC, D and E are points on the sides AB and AC respectively such that DE || BC. If AD = 8cm, AB = 12 cm and AE = 12 cm, then find CE.

In ΔABC, DE || BC.

$\frac{AD}{DB} = \frac{AE}{EC}$



$\frac{8}{4} = \frac{12}{EC}$

8 EC = 4 x 12

$EC = \frac{4 \times 12}{8}$

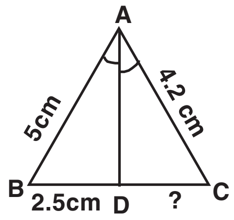
EC = 6cm

4. In ΔABC the internal bisector AD of $\angle A$ meets the side BC at D.

If BD = 2.5 cm. AB = 5 cm and AC = 4.2 cm then find DC. **(Ap. 12, Oct. 12, 13)**

In ΔABC, AD is the internal bisector of $\angle A$

Hence $\frac{AB}{AC} = \frac{BD}{DC}$ (Theorem)



$$\frac{5}{4.2} = \frac{2.5}{DC}$$

$$DC \times 5 = 2.5 \times 4.2$$

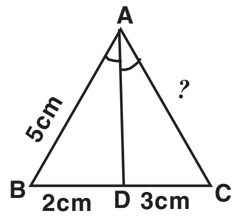
$$DC = \frac{2.5 \times 4.2}{5} = 2.1 \text{ cm}$$

$$DC = 2.1 \text{ cm}$$

5. In a ΔABC, AD is the internal bisector of $\angle A$, meeting BC at D. If BD = 2 cm, AB = 5 cm, DC = 3 cm find AC.

In ΔABC,

AD is the internal bisector of $\angle A$.



$$\frac{AB}{AC} = \frac{BD}{DC}$$

$$\frac{5}{AC} = \frac{2}{3}$$

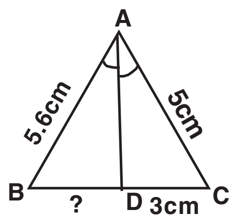
$$2 \times AC = 3 \times 5$$

$$AC = \frac{3 \times 5}{2} = 7.5 \text{ cm}$$

$$AC = 7.5 \text{ cm}$$

6. In ΔABC, AD is the internal bisector of $\angle A$, meeting BC at D. If AB = 5.6 cm, AC = 6 cm and DC = 3 cm find BC.

AD is the internal bisector of $\angle A$.



$$\frac{BD}{DC} = \frac{AB}{AC}$$

$$\frac{BD}{3} = \frac{5.6}{6}$$

$$6 \times BD = 5.6 \times 3$$

$$BD = \frac{5.6 \times 3}{6} = 2.8\text{cm}$$

BD = 2.8 cm

BC = BD + DC

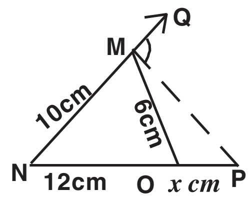
BC = 2.8 + 3 cm

= 5.8 cm

BC = 5.8 cm

7. In a ΔMNO, MP is the external bisector of $\angle M$ meeting NO produced at P. If MN = 10 cm, MO = 6 cm, NO = 12 cm then find OP. (July 13, Oct. 14)

Given that in ΔMNO, MP is the external bisector of $\angle M$ and Let OP = x.



$$\frac{MN}{MO} = \frac{NP}{OP}$$

Let OP = x

PN = PO + ON = x + 12

$$\frac{x + 12}{x} = \frac{10}{6}$$

$$6(12+x) = 10 \times x$$

$$6x + 72 = 10x$$

$72 = 10x - 6x$

$72 = 4x$

$4x = 72$

$x = \frac{72}{4} = 18$

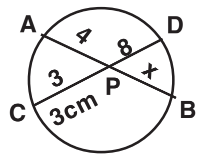
OP = 18 cm

8. Find the value of *x* in the given diagram.

AB, CD are chords

PA x PB = PC x PD

$4 \times x = 3 \times 8$



$x = \frac{3\times8}{4}$

$x = 6$

$x = 6$

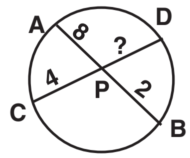
9. AB and CD are two chords of a circle which intersect each other internally at P. If CP = 4 cm, AP = 8 cm, PB = 2 cm, then find PD. **(Apr. 14)**

Given that the chords AB and CD intersect at P, inside the circle.

AP x PB = CP x PD.

8 x 2 = 4 x PD

4 x PD = 8 x 2



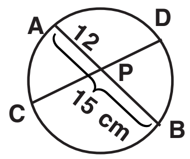
$PD = \frac{8\times2}{4} = 4$

PD = 4cm

10. AB and CD are two chords of a circle which intersect each other internally at P. If AP = 12 cm, AB = 15cm CP = PD then find CD.

AP + PB = 15 cm

12 + PB = 15 cm



PB = 15 - 12

= 3

CP = PD

AB and CD are the chords which interesect internally

PA x PB = PC x PD

12 x 3 = PC x PC [PC = PD]

$36 = PC^2$

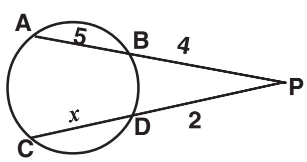
$PC^2 = 36$

$PC = \sqrt{36} = 6$

CD = PC + PD = 6 + 6 = 12 cm

CD = 12 cm

11. Find the value of *x* in the following diagram.



The chords AB and CD intersect externally at P.

PA x PB = PC x PD

9 x 4 = (2 + x) x 2

(2 + x) x 2 = 9 x 4

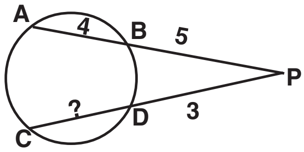
$$2 + x = \frac{9 \times 4}{2}$$

2 + x = 18

x = 18 - 2 = 16

x = 16

12. AB and CD are two chords of a circle which intersect each other externally at P. If AB = 4cm, BP = 5 cm and PD = 3 cm then find CD.



Let CD = x cm

PA x PB = PC x PD

(4 + 5) x 5 = (x + 3) x 3

9 x 5 = 3x + 9

3x + 9 = 45

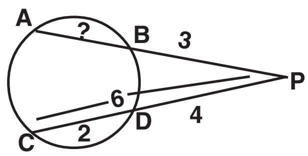
3x = 45 - 9

3x = 36

$$x = \frac{36}{3} = 12$$

CD = 12 cm

13. AB and CD are two chords of a circle which intersect each other externally at P. If BP = 3cm, CP = 6cm and CD = 2 cm then find AB.



Let AB = x cm

PA x PB = PC x PD

(x + 3) x 3 = (2+4) x 4

3x x 9 = 6 x 4

3x + 9 = 24

3x = 24 - 9

3x = 15

$$x = \frac{15}{3} = 5$$

AB = 5 cm

7. TRIGONOMETRY

1. A kite is flying with a string of length 200 m. If the thread makes an angle 30° with the ground, find the distance of the kite from the ground level.

BC = height = x m

AC = length of thread = 200 m

θ = 30°

In ΔABC

$$\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}}$$

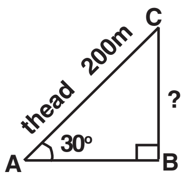
$\sin 30^\circ = \frac{x}{200}$

$\frac{1}{2} = \frac{x}{200}$

$2 \times x = 1 \times 200$

$x = \frac{200}{2} = 100$

$x = 100$ m



Distance of the kite from the ground = 100m

2. A ladder leaning against a vertical wall, makes an angle of 60° with the ground. The foot of the ladder is 3.5 m away from the wall. Find the length of the ladder. **(Oct 12, Apr. 13, June 14)**

Let the length of the ladder be x m

AB = 3. 5 cm

$\angle$BAC = 60°

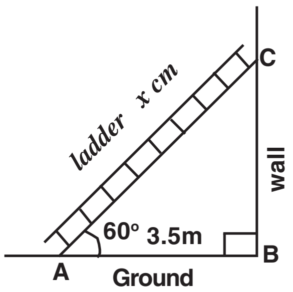
$\cos 60^\circ = \frac{\text{adjacent side}}{\text{hypotenuse}}$

$\cos 60^\circ = \frac{3.5}{x}$

$\frac{1}{2} = \frac{3.5}{x}$

$1 \times x = 2 \times 3.5$

$x = 7$m

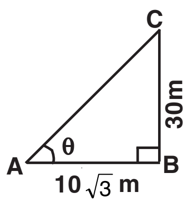


Length of the ladder = 7m.

3. Find the angular elevation of the sun when the length of the shadow of a 30m long pole is $10\sqrt{3}$ m. **(Mar. 12, Mar. 14)**

BC = length of the pole = 30 m

AB = length of the shadow = $10\sqrt{3}$ m, θ = ?



$\tan\theta = \frac{\text{opposite side}}{\text{adjacent side}}$

$\tan\theta = \frac{30}{10\sqrt{3}}$

$= \frac{3}{\sqrt{3}} = \frac{\sqrt{3}.\sqrt{3}}{\sqrt{3}}$

$\tan\theta = \sqrt{3}$

$\tan 60 = \sqrt{3}$ Hence $\theta = 60^\circ$

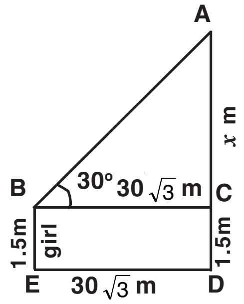
The angular elevation of the sun from the ground level is 60°.

4. The angle of elevation of the top of a tower as seen by an observer is 30°. The observer is at a distance of $30\sqrt{3}$ m from the tower. If the eye level of the observer is 1.5 m above the ground level, then find the height of the tower.

AD = height of the tower = x + 1.5 m

BC = DE = $30\sqrt{3}$ m

In ΔABC $\angle ABC = 30^\circ$



$$\tan\theta = \frac{\text{opposite side}}{\text{adjacent side}}$$

$$\tan\theta = \frac{x}{30\sqrt{3}}$$

$$\frac{1}{\sqrt{3}} = \frac{x}{30\sqrt{3}}$$

$$x\sqrt{3} = 30\sqrt{3}$$

$$x = \frac{30\sqrt{3}}{\sqrt{3}} = 30$$

$$x = 30 \text{ m}$$

Height of the tower = x + 1.5 cm

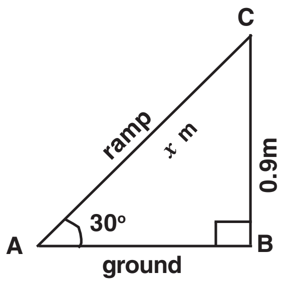
= 30 + 1.5 = 31. 5 m

5. A ramp for unloading a moving truck, has an angle of elevation of 30°. If the top of the ramp is 0.9 m above the ground level, then find the length of the ramp. **(Oct. 14, Mar. 15)**

AC = length of the ramp = xm

BC = 0.9m

$\angle CAB = 30^\circ$



$$\sin q = \frac{\text{adjacent side}}{\text{hypotenuse}}$$

$$\sin 30^\circ = \frac{0.9}{x}$$

$$\frac{1}{2} = \frac{0.9}{x}$$

$1 \times x = 0.9 \times 2$

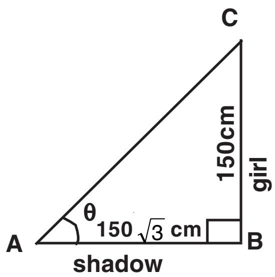
x = 1.8 m

Length of the ramp is 1.8 m.

6. A girl of height 150 cm stands in front of a lamp-post and casts a shadow of length 150 $\sqrt{3}$ cm on the ground. Find the angle of elevation of the top of the lamp-post. **(June 12)**

AB = height of the girld = 150 cm

BC = length of the shadow = 150 $\sqrt{3}$ cm, θ = ?



$$\tan\theta = \frac{\text{opposite side}}{\text{adjacent side}}$$

$$\tan\theta = \frac{150}{150\sqrt{3}}$$

$$\tan\theta = \frac{1}{\sqrt{3}}$$

$$\tan 30^\circ = \frac{1}{\sqrt{3}}$$

$\tan 30^\circ = \frac{1}{\sqrt{3}}$. Hence $\theta = 30^\circ$

The angle of elevation of the top of the lamp-post = 30°.

7. Prove the identity $\frac{\sin\theta}{\operatorname{cosec}\theta}+\frac{\cos\theta}{\sec\theta}$ = 1 **(June 12)**

$$\textbf{LHS} = \frac{\sin\theta}{\operatorname{cosec}\theta}+\frac{\cos\theta}{\sec\theta}$$

$$= \frac{\sin\theta}{\left(\frac{1}{\sin\theta}\right)}+\frac{\cos\theta}{\left(\frac{1}{\cos\theta}\right)}$$

$$= \frac{\sin\theta.\sin\theta}{1}+\frac{\cos\theta.\cos\theta}{1}$$

$= \sin^2\theta + \cos^2\theta$

= 1

= **RHS**

8. Prove that $\sqrt{\frac{1-\sin\theta}{1+\sin\theta}}$ = secθ - tanθ. **(Oct. 12, June 14)**

$$\textbf{LHS} = \sqrt{\frac{1-\sin\theta}{1+\sin\theta}}$$

$$= \sqrt{\frac{1-\sin\theta}{1+\sin\theta}\times\frac{1-\sin\theta}{1-\sin\theta}} \quad \text{(Multiplied by the conjugate)}$$

$$= \sqrt{\frac{(1-\sin\theta)^2}{1-\sin^2\theta}}$$

$$= \sqrt{\frac{(1-\sin\theta)^2}{\cos^2\theta}}$$

$$= \sqrt{\left(\frac{1-\sin\theta}{\cos\theta}\right)^2}$$

$$= \frac{1-\sin\theta}{\cos\theta}$$

$$= \frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}$$

= secθ - tanθ

= **RHS.** Hence the result.

9. Prove the identity $\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$ = cosecθ - cotθ

$$\textbf{LHS} = \sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = \sqrt{\frac{(1-\cos\theta)\,(1-\cos\theta)}{(1+\cos\theta)(1-\cos\theta)}} \quad \text{(Multiplied by the conjugate)}$$

$$= \sqrt{\frac{(1-\cos\theta)^2}{1^2-\cos^2\theta}}$$

$$= \sqrt{\frac{(1-\cos\theta)^2}{\sin^2\theta}}$$

$$= \sqrt{\left(\frac{1-\cos\theta}{\sin\theta}\right)^2}$$

$$= \frac{1-\cos\theta}{\sin\theta}$$

$$= \frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}$$

= cosecθ - cotθ

= **RHS** Hene the result.

10. Prove that $\frac{\cos\theta}{\sec\theta - \tan\theta}$ = 1 + sinθ. **(June 13)**

LHS $= \frac{\cos\theta}{\sec\theta - \tan\theta}$

$$= \frac{\cos\theta}{\left(\frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}\right)}$$

$$= \frac{\cos\theta}{\left(\frac{1-\sin\theta}{\cos\theta}\right)}$$

$$= \frac{\cos\theta . \cos\theta}{1-\sin\theta}$$

$$= \frac{\cos^2\theta}{1-\sin\theta} = \frac{1-\sin^2\theta}{1-\sin\theta} = \frac{1^2-\sin^2\theta}{1-\sin\theta}$$

$$= \frac{(1+\sin\theta)(1-\sin\theta)}{(1-\sin\theta)}$$

= 1 + sinθ

= **RHS**

11. Prove the identity $\frac{\sin\theta}{\cos ec\theta + \cot\theta}$ = 1 - cosθ **(Oct. 14)**

LHS $= \frac{\sin\theta}{\cos ec\theta + \cot\theta}$

$$= \frac{\sin\theta}{\left(\frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta}\right)}$$

$$= \frac{\sin\theta}{\left(\frac{1+\cos\theta}{\sin\theta}\right)}$$

$$= \frac{\sin\theta . \sin\theta}{1+\cos\theta}$$

$$= \frac{\sin^2\theta}{1+\cos\theta} = \frac{1-\cos^2\theta}{1+\cos\theta} = \frac{1^2-\cos^2\theta}{1+\cos\theta}$$

$= \frac{(1+\cos\theta)(1-\cos\theta)}{(1+\cos\theta)}$

$= 1 - \cos\theta$

= **RHS**

12. Prove the identity $\sqrt{\sec^2\theta + \text{cosec}^2\theta}$ = tanθ + cotθ **(Mar. 14, Mar. 15)**

LHS $= \sqrt{\sec^2\theta + \text{cosec}^2\theta}$ $\sec^2\theta = 1 + \tan^2\theta$

$= \sqrt{(1+\tan^2\theta) + (1+\cot^2\theta)}$ $\text{cosec}^2\theta = 1 + \cot^2\theta$

$= \sqrt{\tan^2\theta + 2 + \cot^2\theta}$ $\tan\theta . \cot\theta = \tan\theta \times \frac{1}{\tan\theta}$

$= \sqrt{\tan^2\theta + 2\tan\theta.\cot\theta + \cot^2\theta}$ $a^2 + 2ab + b^2 = (a+b)^2$

$= \sqrt{(\tan\theta + \cot\theta)^2}$

$= \tan\theta + \cot\theta$

= **RHS**

13. Prove that $\frac{1+\sec\theta}{\sec\theta} = \frac{\sin^2\theta}{1-\cos\theta}$ **(Oct. 13)**

L.H.S. $= \frac{1+\sec\theta}{\sec\theta}$

$= \frac{\left(1+\frac{1}{\cos\theta}\right)}{\left(\frac{1}{\cos\theta}\right)}$

$= \frac{\left(\frac{\cos\theta+1}{\cos\theta}\right)}{\left(\frac{1}{\cos\theta}\right)}$

$= \frac{(\cos\theta+1) \times \cos\theta}{\cos\theta \times 1} = 1 + \cos\theta$

$= (1 + \cos\theta)\frac{(1-\cos\theta)}{(1-\cos\theta)}$ (Multiplied both Nr & Dr by conjugate)

$= \frac{1-\cos^2\theta}{1-\cos\theta}$

$= \frac{\sin^2\theta}{1-\cos\theta}$

= R.H.S.

14. Prove that $(\sin^6\theta + \cos^6\theta) = 1 - 3\sin^2\theta\cos^2\theta$ **(Mar. 12)**

LHS $= \sin^6\theta + \cos^6\theta$ $a^3 + b^3 = (a+b)^3 - 3ab(a+b)$

$= (\sin^2\theta)^3 + (\cos^2\theta)^3$

$= (\sin^2\theta + \cos^2\theta)^3 - 3\sin^2\theta\cos^2\theta(\sin^2\theta + \cos^2\theta)$

$= (1)^3 - 3\sin^2\theta.\cos^2\theta \times 1$

$= 1 - 3\sin^2\theta.\cos^2\theta$

= **RHS**

15. Prove that $\frac{\sin\theta}{1-\cos\theta}$ = cosecθ + cotθ.

L.H.S. $= \frac{\sin\theta}{1-\cos\theta}$

$= \frac{\sin\theta(1+\cos\theta)}{(1-\cos\theta)(1+\cos\theta)}$

$= \frac{\sin\theta(1+\cos\theta)}{1-\cos^2\theta}$

$= \frac{(1+\cos\theta)}{\sin\theta}$

$= \frac{1}{\sin\theta}+\frac{\cos\theta}{\sin\theta}$

= cosecθ + cotθ

= RHS

16. Prove the following identities.

i) $\sec^2\theta + \text{cosec}^2\theta = \sec^2\theta\,\text{cosec}^2\theta$

vi) $\frac{1+\cos\theta-\sin^2\theta}{\sin\theta(1+\cos\theta)} = \cot\theta$

vii) $\sec\theta\,(1-\sin\theta)\,(\sec\theta + \tan\theta) = 1$

Try yourself using the above steps.

8. MENSURATION

1. A solid right circular cylinder has radius 7 cm and height 20 cm. Find its i) curved surface area ii) total surface area. (Take $\pi = 22/7$)

Solution :

Given that r = 7 cm and h = 20 cm

i) Curved surface area of cylinder $= 2\pi rh$

$= 2 \times \frac{22}{7} \times 7 \times 20$

= 880 sq. cm

ii) Total surface area of cylinder $= 2\pi r\,(h+r)$

$= 2 \times \frac{22}{7} \times 7 \times (20+7)$

= 44 x 27

= 1188 sq.cm

2. A solid right circular cylinder has radius 14 cm and height 8 cm. Find its curved surface area and total surface area.

Solution :

Given that r = 14 cm and h = 8 cm

Curved surface area of cylinder $= 2\pi rh$

$= 2 \times \frac{22}{7} \times 14 \times 8$

= 704 sq. cm

Total surface area of cylinder $= 2\pi r\,(h+r)$

$= 2 \times \frac{22}{7} \times 14 \times (8+14)$

$= 2 \times \frac{22}{7} \times 14 \times 22$

$= 1936\ cm^2$.

3. Find the volume of a solid cylinder whose radius is 14 cm and height 30 cm.

Solution:

Given that r = 14cm and h = 30 cm

Volume of a cylinder $= \pi r^2 h$

$= \frac{22}{7} \times 14 \times 14 \times 30$

$= 18480\ cm^3$

4. A patient in a hospital is given soup daily in a cylindrical bowl of diameter 7 cm. If the bowl is filled with soup to a height of 4 cm, then find the quantity of soup to be prepared daily in the hospital to serve 250 patients?

Solution:

Given that 2r= 7 cm and h = 4 cm

$\therefore\ r = \frac{7}{2}$ cm

The quantity of soup for one patient $= \pi r^2 h$

$= \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 4$

$= 154\ cm^3$

Quantity of soup needed for 25 patients $= 250 \times 154$

$= 38500\ cm^3 = \frac{38500}{1000}$ litres

= 38.5 litres   1 litre = 1000 cm^3

5. Volume of a solid cylinder is 62.37 cu.cm. Find the radius if its height is 4.5 cm.

Solution:

Given that h = 4.5 cm

Volume of a solid cylinder = 62.37 cu.cm

i.e. $\pi r^2 h = 62.37\ cm^3$

$r^2 = \frac{62.37}{\pi h}$

$= 62.37 \times \frac{7}{22} \times \frac{1}{4.5}$

$= 4.41$

$r = \sqrt{4.41} = 2.1$ cm

6. The radii of two right circular cylinders are in the ratio of 2 : 3. Find the ratio of their volumes if their heights are in the ratio 5 : 3.

Solution :

Given that $r_1, r_2 = 2 : 3$ and $h_1, h_2 = 5 : 3$

Let $r_1 = 2x$ and $r_2 = 3x$, $h_1 = 5y$ and $h_2 = 3y$

Ratio of the volume of the two cylinders

$= \pi r_1^2 h_1 : 2\pi r_2^2 h_2$

$= r_1^2 h_1 : r^2_2 h_2$

$= 2x \times 2x \times 5y : 3x \times 3x \times 3y$

$= 20 : 27$

7. Radius and slant height of a solid right circular cone are 35 cm and 37 cm respectively. Find the curved surface area and total surface area of the cone. (Take $\pi = \frac{22}{7}$)

Solution :

Given that r = 35 cm, l = 37 cm

Curved surface area = πrl

$= \frac{22}{7} \times 35 \times 37$

= 4070 sq.cm

Total surface area $= \pi r (l+r)$

$= \frac{22}{7} \times 35 (37 + 35)$

$= \frac{22}{7} \times 35 \times 72$

= 7920 sq.cm

8. If the circumference of the base of a solid right circular cone is 236 cm and its slant height is 12 cm, find its curved surface area.

Solution:

Circumference of the cone = 236 cm, ℓ = 12 cm

ie $2\pi r$ = 236 cm

$\therefore \pi r$ = 118 cm

Curved surface are $= \pi r\ell$

= 118 x 12

= 1416 cm^2

9. The circumference of the base of a 12 m high wooden solid cone is 44 m. Find the volume.

Solution:

Given that circumference = 44 cm and h = 12 cm

ie $2\pi r = 44$

$\pi r = 22$

$r = \frac{22}{\pi}$

$= \frac{22 \times 7}{22}$

r = 7 cm

Volume of the bowl $= \frac{1}{3} \pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times 12$

$= 616m^2$

10. The volume of a cone with circular base is 216π cu.cm. If the base radius is 9 cm, then find the height of the cone.

Solution:

Volume of the cone = 216π cu.cm and r = 9 cm

$$\text{i.e. } \frac{1}{3}\pi r^2 h = 216\pi$$

$$\frac{1}{3}\pi \times 9 \times 9 \times h = 216\pi$$

$$h = \frac{216 \times 3}{9 \times 9} = 8$$

height = 8 cm

11. Find the volume of the largest right circular cone that can be cut out of a cube whose edge is 14 cm.

Solution:

Given that side of the cube =14 cm

$\therefore$ radius of the cone = $\frac{14}{2}$ = 7 cm

height of the cone = 14 cm

$\therefore$ Volume of the cone = $\frac{1}{3}\pi r^2 h$

$$= \frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times 14$$

= 718.67 cm^3

12. The radii of two circular ends of a frustum shaped bucket are 15 cm and 8 cm. If its depth is 63cm. Find the capacity of the bucket in litres. (Take $\pi = \frac{22}{7}$)

Solution:

Given that R = 15cm, r = 8 cm and h = 63cm

The volume of the bucket = $\frac{1}{3}\pi h (R^2 + r^2 + Rr)$

$$= \frac{1}{3} \times \frac{22}{7} \times 63 \times (15^2 + 8^2 + 15 \times 8)$$

$$= \frac{1}{3} \times \frac{22}{7} \times 63 \times (225 + 64 + 120)$$

$$= \frac{1}{3} \times \frac{22}{7} \times 63 \times 409$$

= 26994 cu.cm

$$= \frac{26994}{1000} \text{ litres}$$

= 26.994 litres

13. A hollow sphere in which a circus motorcyclist performs his stunts, has an inner diameter of 7 m. Find the area available to the motorcylist for riding. (Take $\pi = \frac{22}{7}$)

Solution:

Given that diameter = 7m $\Rightarrow$ r = 7/2 m

Area to the motorcyclist for riding = $4\pi r^2$

$$= 4 \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}$$

= 154 sq.m.

14. The thickness of a hemispherical bowl is 0.25 cm. The inner radius of the bowl is 5 cm. Find the outer curved surface area of the bowl. (Take $\pi = \frac{22}{7}$)

Solution:

Given that w = 0.25 cm , r = 5 cm

$\therefore$ R = r + w

= 5 + 0.25 = 5.25

$\therefore$ Outer surface area of the bowl $= 2\pi R^2$

$$= 2 \times \frac{22}{7} \times 5.25 \times 5.25$$

= 173.25 sq.cm.

15. If the curved surface area of a solid sphere is 98.56 cm^2, then find the radius of the sphere.

Solution :

Given that curved surface area = 98.56cm^2

ie $4\pi r^2 = 98.56$

$$4 \times \frac{22}{7} \times r^2 = 98.56$$

$$r^2 = \frac{98.56 \times 7}{4 \times 22} = 7.84$$

$$r = \sqrt{7.84} = 2.8 \text{ cm}$$

r = 2.8 cm

16. Find the volume of a sphere shaped metallic shot-put having diameter of 8.4 cm. (Take $\pi = \frac{22}{7}$)

Solution:

Given that 2r = 8.4 cm $\Rightarrow$ r = 4.2 cm

Volume of the shot-put $= \frac{4}{3}\pi r^3$

$$= \frac{4}{3} \times \frac{22}{7} \times 4.2 \times 4.2 \times 4.2$$

= 310.464 cu.cm

17. The outer and the inner radii of a hollow sphere are 12 cm and 10 cm. Find its volume.

Solution:

Given that R = 12 cm and r = 10 cm

$$\text{Volume} = \frac{4}{3}\pi (R^3 - r^3)$$

$$= \frac{4}{3} \times \frac{22}{7} (12^3 - 10^3)$$

$$= \frac{4}{3} \times \frac{22}{7} (1728 - 1000)$$

$$= \frac{4}{3} \times \frac{22}{7} \times 728$$

= 3050 .66 cu.cm.

18. The volume of a solid hemisphere is 1152π cu.cm. Find its curved surface area.

Solution:

Given that volume of the hemisphere = 1152π

$$\text{ie } \frac{2}{3}\pi r^3 = 1152\,\pi$$

$$r^3 = \frac{1152 \times 3}{2}$$

$$= 1728$$

$$r = \sqrt[3]{1728} = 12 \text{ cm}$$

Curved surface area = $2\pi r^2 = 2 \times \pi \times 144 = 288\pi cm^2$.

11. STATISTICS

1. Find the range and coefficient of range for 43, 24, 38, 56, 22, 39, 45.

L = 56, S = 22

i) Range = L - S = 56 - 22 = 34

= 34

ii) Coefficient of range = $\frac{L-S}{L+S} = \frac{34}{78} = 0.436$

2. Find the range and coefficient of range of the following data 59, 46, 30, 33, 27, 40, 52,35, 29**.**

L = 59 , S = 23

Range = L - S

= 59 - 23 = 36

Coefficient of range = $\frac{L-S}{L+S} = \frac{36}{82}$

= 0.44

3. The largest value in a collection of data is 7.44. If the range is 2.26, Find the smallest value of data.

Range = L - S   Range = 2.26 and L = 7.44

2.26 = 7.44 - S $\Rightarrow$ S = 7.44 - 2.26

= 5.18

4. Smallest value of data is 12. The range is 59. Find the largest value of data.

Range = L - S   Range = 59 and S = 12

59 = L - 12 $\Rightarrow$ L = 59 + 12 = 71

5. Largest of 50 measurements is 3.84kg. If the range is 0.46kg, find the smallest value of measurement?

Range = L - S   L = 3.84   Range = 0.46

0.46 = 3.84 - S $\Rightarrow$ S = 3.84 - 0.46 = 3.38 Kg

6. Find standard deviation for first 10 natural numbers.

Standard deviation for first n natural numbers $= \sqrt{\frac{n^2-1}{12}}$; n = 10

$$= \sqrt{\frac{10^2-1}{12}} = \sqrt{\frac{100-1}{12}} = \sqrt{\frac{99}{12}} \simeq 2.87$$

7. Find Standard deviation for first 13 natural numbers.

Standard deivation for first n natural numbers = $\sqrt{\frac{n^2-1}{12}}$; *n = 13*

$$= \sqrt{\frac{13^2-1}{2}} = \sqrt{\frac{169-1}{12}} = \sqrt{\frac{168}{12}} = \sqrt{14} \simeq 3.74$$

8. If the coefficient of variation of a collection of data is 57 and its SD is 6.84, then find the mean.

From the given data

$$C.V = \frac{\sigma}{\bar{x}} \times 100\ \%, \quad C.V. = 57, \quad \sigma = 6.84$$

$$57 = \frac{6.84}{\bar{x}} \times 100 \Rightarrow \bar{x} = \frac{684}{57} = 12$$

9. n = 10 $\bar{x} = 12$ $\sum x^2 = 1530$, find the co-efficient of variation.

$$\sigma^2 = \frac{\sum x^2}{n} - (\bar{x})^2 = \frac{1530}{10} - (12)^2$$

$$= 153 - 144 = 9$$

$$\sigma = \sqrt{9} = 3$$

$$C.V. = \frac{\sigma}{\bar{x}} \times 100\ \%$$

$$C.V. = \frac{3}{12} \times 100 = 25\%$$

12. PROBABILITY

1. An integer is chosen from the first twenty natural numbers. What is the probability that it is a prime number?

S = {1, 2, 3, ... 20}, ie. n (S) = 20

Prime number A = {2, 3, 5, 7, 11, 13, 17, 19}, n (A) = 8

$$P(A) = \frac{n(A)}{n(S)} = \frac{8}{20} = \frac{2}{5}$$

2. There are 7 defective items in a sample of 35 items. Find the probability that an item chosen at random is non-defective.

n(S) = 35

No. of defective items = 7

No. of non-defective = 35 – 7 = 28, n(A) = 28

$$P(A) = \frac{28}{35} = \frac{4}{5}$$

3. There are 20 boys and 15 girls in a class of 35 students . A student is chosen at random. Find the probability that the chosen student is a (i) boy (ii) girl.

n(S) = 35

i) No. of boy : A, n(A) = 20

$$P(A) = \frac{20}{35} = \frac{4}{7}$$

ii) No. of girl : B, n(B) = 15

$$P(B) = \frac{15}{35} = \frac{3}{7}$$

4. The probability that it will rain on a particular day is 0.76. What is the probability that it will not rain on that day?

$$P(A) + P(\overline{A}) = 1$$

$$P(\overline{A}) = 1 - 0.76 = 0.24$$

5. Find the probability that a non-leap year selected will have 53 Fridays.

Non leap year have 52 weeks (52 fridays) } = 365 days = 52 weeks + 1 day

1 day contains { Sun, Mon, Tue, Wed, Thu, Fri, Sat}

n(S) = 7

A = {Fri}, n(A) = 1, $P(A) = \frac{1}{7}$

6. Find the probability that a leap year selected at random will have 53 Fridays.

Leap year have 52 Weeks (52 Fridays) } = 366 days = 52 weeks + 2 days

2 days contain = {(Sun, Mon) (Mon, Tue) (Tue, Wed) (Wed, Thu) (Thu, Fri), (Fri, Sat), (Sat, Sun)}

n (S) = 7

A = {(Thu, Fri) (Fri, Sat)}

n (A) = 2

$$P(A) = \frac{2}{7}$$

7. A ticket is drawn from a bag containing 100 tickets. The tickets are numbered from 1 to 100. What is the probability of getting a ticket with a number divisible by 10?

n(S) = 100

Multiples of 10 : A = {10, 20, 30, 40, 50, 60, 70, 80, 90, 100}

n(A) = 10

$$P(A) = \frac{10}{100} = \frac{1}{10}$$

8. A die is thrown twice. Find the probability of getting a total of 9.

S = {(1, 1) (1, 2) (1,3) (1, 4) (1,5) (1,6) (2, 1) (2,2) (2,3) (2,4) (2,5) (2,6)
(3,1) (3, 2) (3, 3) (3, 4) (3,5), (3,6), (4, 1) (4,2) (4,3) (4,4) (4,5) (4,6)
(5,1) (5,2) (5,3) (5,4) (5,5) (5,6) (6,1) (6,2) (6,3) (6,4) (6, 5) (6, 6)},

n(S) = 36

Total of 9 : A = {(3, 6) (4, 5) (5, 4) (6, 3)}

n(A) = 4

$$P(A) = \frac{4}{36} = \frac{1}{9}$$

9. Three rotten eggs are mixed with 12 good ones. One egg is chosen at random, what is the probability of choosing a rotten egg?

$n(S) = 12 + 3 = 15$

Rotten egg : A $n(A) = 3$

$$P(A) = \frac{3}{15} = \frac{1}{5}$$

10. Two coins are tossed together. What is the probability of getting at most one head.

S = {HH, HT, TH, TT} $n(S) = 4$

Atmost one head: A ; A = {HT, TH, TT}

$n(A) = 3$

$$P(A) = \frac{3}{4}$$

11. A bag contains 6 white balls numbered 1 - 6, 4 red balls numbered 7-10. Find the probability of i) even number of balls ii) white balls

$n(S) = 6 + 4 = 10$

Even numbers Balls : A = {2, 4, 6, 8, 10} ; $n(A) = 5$

$$P(A) = \frac{5}{10} = \frac{1}{2}$$

White balls: B = {1, 2, 3, 4, 5, 6} ; $n(B) = 6$

$$P(B) = \frac{6}{10} = \frac{3}{5}$$

12. 20 cards are numbered from 1 to 20. What is the probability that the number is a multiple of 4

$n(S) = 20$

Multiple of 4 : A = {4, 8, 12, 16, 20}; $n(A) = 5$

$$P(A) = \frac{5}{20} = \frac{1}{4}$$

13. Three dice are thrown. Find the probability of getting the same number on all the three dice.

The sample space is the collection of all possible outcomes.

S = {(1, 1, 1) ...... (6,6, 6)} $n(S) = 6 \times 6 \times 6 = 216$

A = {(1, 1, 1) (2, 2, 2) (3, 3, 3) (4, 4, 4) (5, 5, 5) (6, 6, 6)

$n(A) = 6$

$$P(A) = \frac{6}{216} = \frac{1}{36}$$

14. From a well shuffled pack of 52 cards, one card is drawn at random Find the probability of getting i) Black king card ii) Spade card.

$n(S) = 52$

i) Black king : A; $n(A) = 2$; $P(A) = \frac{2}{52} = \frac{1}{26}$

ii) Spades : B; $n(B) = 13$; $P(B) = \frac{13}{52} = \frac{1}{4}$